THE Art OF ACCOMPLISHMENT

THE NATIONAL ANNUAL 2016 FINANCE AND INVESTMENT MANAGEMENT OLYMPIAD

LEARNER GUIDE

ORGANISER

PROVERTO

SPONSOR

strate

South Africa's Central Securities Depository

RETHINK. REINVENT.

UNIVERSITY OF JOHANNESBURG

TABLE OF CONTENTS

ABOUT THE FINANCE AND INVESTMENT MANAGEMENT OLYMPIAD (FIMO)

The Finance and Investment Management Olympiad (FIMO) aims to introduce the concepts that are fundamental to the finance and investment management related study fields. The University of Johannesburg offers a BCom Finance degree which is a stepping stone towards the following honours degrees:

- BCom Honours Financial Management
- BCom Honours Financial Planning
- BCom Honours Investment Management
- BCom Honours Property Valuation and Management
- BCom Honours Quantitative Finance
- BCom Honours Treasury Management
- Postgraduate Diploma in Estate Planning
- Postgraduate Diploma in Financial Management
- Postgraduate Diploma in Financial Markets

In order to pursue any one of these above mentioned programmes, a BCom Finance degree with related electives needs to be obtained. The entry requirements for the BCom Finance degree are:

- Minimum APS scale of 30
- Language of learning and teaching: minimum rating of 4
- Additional recognised language: minimum rating of 2
- Mathematics: minimum rating of 4 (Mathematical Literacy is not accepted, please refer to the BCom Finance extended degree or the Diplomas offered by the Faculty of Economic and Financial Sciences at the University of Johannesburg)
- Life Orientation: minimum rating of 3
- Group B: minimum rating of 4 for two subjects and a minimum rating of 3 for the third subject

The University of Johannesburg does offer a Diploma in Financial Services Operations as well as multiple Advanced Diplomas which is an alternative route to the above mentioned Honours and Postgraduate Diploma programmes listed above. Alternatively, a similar Degree or Diploma with an Advanced Diploma from another University can also be considered for the Honours and Postgraduate programmes.

The financial services industry in South Africa is comprised of a broad range of institutions. Examples of these institutions are:

- Financial Services Board
- Johannesburg Stock Exchange
- National Treasury
- South African Reserve Bank
- STRATE
- Financial Services Institutions, examples including:
    - Asset management companies
    - Banks (private and retail)
    - Brokerages
    - Insurance companies

Examples of careers in the financial services industry are:

- Certified Management Accountant
- Corporate Financier
- Credit Analyst
- Financial Analyst
- Financial Consultant
- Financial Manager
- Financial Planner

- Financial Modeller
- Financial Risk Manager
- Investment Advisor
- Portfolio Manager
- Private Banker
- Quantitative Finance Analyst
- Stockbroker
- Trader
- Wealth Manager

Examples of national and international professional bodies related to the finance and investment management fields are:

- ACI: The Financial Markets Association (www.acifma.com, http://www.aciforex.co.za)
- ACTSA: Association of Corporate Treasurers of Southern Africa (www.actsa.org.za)
- CAIA Association : Chartered Alternative Investment Analyst (http://caia.org)
- CFA Institute: Chartered Financial Analyst (www.cfainstitute.org)
- CIMA: Chartered Institute of Management Accountants (www.cimaglobal.com)
- CISI: Chartered Institute for Securities & Investment (www.cisi.org)
- FPI: Financial Planning Institute of Southern Africa (www.fpi.co.za)
- GARP: Global Association of Risk Professionals (www.garp.org)
- IOB: Institute of Bankers South Africa (www.iob.co.za)
- MTA: Market Technicians Association (http://www.mta.org)
- PRMIA: Professional Risk Managers' International Association (www.prmia.org)
- SACPVP: South African Council for the Property Valuers Profession (www.sacpvp.co.za)
- SAIS: South African Institute of Stockbrokers (www.sais.co.za)
- Society of Actuaries – Investment Track (www.soa.org)

The current high school curriculum for schools in South Africa, do not include Financial Management and/or Investment Management as subjects. However, Financial Management and Investment Management at a higher education curriculum level is a combination of Accounting, Economics, Business Studies and Math.

The topic areas which form part of the Finance and Investment Management Olympiad include:

- Economics
- Financial Literacy: Saving and borrowing
- Financial Markets and Instruments
- Financial Math
    - Interest Rates
    - Risk and Return
    - Time value of money
- Financial Statements
- Financial System
- Math

An understanding of Grade 10 Economics, Accounting and Math is required for the Olympiad; however it will not be included in the provided study material. If you need assistance with Economics and Financial Statements, please refer to Investopedia: http://www.investopedia.com/

INTRODUCTION

Finance and investment management is a broad area made up of many aspects and levels related to the financial services industry. The core grounding of finance and investment management is made of up a financial system which consists of the following components:

- Financial participants
- Financial markets
- Financial instruments
- Investments:
    - Risk and return
- Financial Mathematics:
    - Interest rates
    - Time value of money

These concepts are the core building blocks which need to be understood in order to advance in the finance and investment management environment. It is important to understand that the above-mentioned concepts do not cover all components of finance and investment management, but it provides a solid foundation.

A financial system is a set of arrangements / conventions embracing the lending and borrowing of funds to non-financial economic units and the intermediation (linking) of this function by financial intermediaries in order to facilitate the transfer of funds, to create additional money when required, and to create markets in debt and equity instruments (and their derivatives) so that the price and allocation of funds are determined efficiently.

The financial system is made up of six important components:

1. Lenders and borrowers: non-financial economic units that undertake the lending and borrowing process
2. Financial intermediaries: intermediate the lending and borrowing process, they are the link between lenders and borrowers; i.e. banks, insurance companies and asset managers
3. Financial instruments: created to satisfy the needs of the financial participants
4. Creation of money, when required: unique money creating ability of banks (different interest rates between borrowers and savers [lenders])
5. Financial markets: institutional arrangements and conventions that exist for the issue and trading [dealing] of financial instruments
6. Price discovery: determination or making of the price of equity and the price of money / debt [rate of interest]

All the *ultimate lenders and borrowers* (household, corporate, government and foreign sectors) and all the financial *intermediaries* are participants in the financial markets. Additional participants are the *other financial entities* that facilitate the transfer of funds and securities: the broker-dealers, the regulators, the financial exchanges (which essentially do no more that facilitate the transfer of securities) and fund managers.

Additional participants which make use of the financial system are made up of the following:

- Brokers and dealers: members of exchanges and / or financial intermediaries that facilitate the trade in financial instruments (also referred to as broker-dealers)
- Fund managers (portfolio managers): corporate entities / departments of financial intermediaries that manage funds on behalf of principals (owners or holders of money)
- Financial exchanges: allow the broker-dealers to facilitate trading in securities and create the mechanism for clearing and settlement of trades in risk-minimising manner
- Credit rating agencies: analyse relevant financial and economic data pertaining to the issuers of securities and assign ratings to the securities reflecting the probability of the issuers meeting their financial obligation (made up of interest and principal)
- Financial regulators: regulate and supervise all players in the financial system

In terms of its main economic function, the financial markets provide *channels for transferring the excess funds of surplus units to deficit units*. Financial markets thus constitute *the mechanism that links surplus and deficit units*, providing the means for surplus units to finance deficit units either directly or indirectly through financial intermediaries. An allied and vital function is price discovery.

The financial markets are simply the mechanisms and conventions that exist for the transfer of funds and their counterparts (i.e. the financial instruments) between the various participants.

CHAPTER 1: SOUTH AFRICA'S FINANCIAL SERVICES SECTOR

The South African Financial Services sector comprises of Banking and Finance, Insurance, Professional Services, and Governmental Corporations and Parastatals. Professional Services can be broken down into Financial Intermediation and Accountancy. The range of services offered include: commercial, retail and merchant banking, mortgage lending, insurance and investment.

21.1% of South Africa's gross domestic product (GDP) is attributed to the finance, real estate and business services sector; the sector has historically added to real economic growth, despite overall growth being negative. The South African financial services sector is internationally recognised as having an extremely strong legal and regulatory framework, and is at the forefront of financial markets within Africa.

The central bank of South Africa, the South African Reserve Bank (SARB) (https://www.resbank.co.za/Pages/default.aspx), has the jurisdiction to set and implement monetary policy, as well as control the domestic interest rate environment. The SARB aims at facilitating sustainable economic development and growth through efficient and effective economic policies, whilst maintaining price stability. The SARB manages and governs the financial services sector, whilst the Financial Services Board (FSB) has jurisdiction over the non-banking financial services industry.

The FSB (https://www.fsb.co.za/Pages/Home.aspx) is an independent body that is fully funded by fees and levies charged within the industry. The board aims at maintaining stability within the industry and protecting financial consumers, through its establishment and maintenance of regulatory frameworks. The FSB simultaneously works with other boards throughout Africa to establish comprehensive regulatory guidelines.

The South African stock exchange (JSE) (https://www.jse.co.za/about/history-company-overview), situated in Johannesburg, is the largest in Africa and ranked nineteenth globally, by market capitalisation. Since its formation in 1887, the JSE has developed into an electronic trading system that offers investors exposure to the leading capital markets locally and throughout Africa. Settlements of all equities and bonds transactions are facilitated by Strate.

Strate (http://www.strate.co.za/about-strate/our-company) is a central securities depository (CSD), internationally recognised as a financial market infrastructure (FMI), whose aim is to support and promote the safety and efficiency of the financial markets. In addition to providing electronic settlement of equities and bonds transactions for the JSE, Strate settles money market security transactions and has recently introduced a collateral management service.

The South African financial services sector is a comprehensively regulated and efficiently managed segment of the South African economy. Comprising of both domestic and international institutions, offering a kaleidoscope of services, the sector is recognised globally for its efficiency and sound regulatory framework.

CHAPTER 2: JSE CODE OF ETHICS AND STANDARDS OF PROFESSIONAL CONDUCT

CODE OF ETHICS AND STANDARDS OF PROFESSIONAL CONDUCT APPLICABLE TO SPONSORS, DESIGNATED ADVISORS AND DEBT SPONSORS

PREAMBLE

The Johannesburg Stock Exchange ("JSE") Code of Ethics and Standards of Professional Conduct ("Code and Standards") is essential for the maintenance of exceptional regulation in the listed environment. All sponsors, designated advisors, debt sponsors and their approved executives ("Sponsors and Executives") must adhere to the Code and Standards.

CODE OF ETHICS

Sponsors and Executives should, in the context of the JSE sponsor function, exercise the utmost integrity, competence, diligence, and confidentiality in their dealings with the JSE, their clients and prospective clients, employers and colleagues. The following fundamental principles should be applied:

A. **Integrity and Objectivity.** Sponsors and Executives should remain transparent and honest in all professional and business relationships and should not allow bias, conflict of interest or the undue influence of others to override their professional judgement.

B. **Professional Competence and Due Care.** Sponsors and Executives have an ongoing duty to maintain their professional knowledge and skill at such a level as to ensure that their clients receive competent and professional service in line with up-to-date developments in professional and best practice, legislation and the Listings Requirements. Sponsors and Executives should act diligently and in accordance with applicable technical and professional standards when rendering professional services.

C. **Confidentiality.** Sponsors and Executives should respect the confidential nature of information acquired in the context of professional and business relationships. Such confidential information may not be used by Sponsors and Executives for personal gain and should not be disclosed to third parties without due authority or unless there exists a legal obligation of disclosure.

STANDARDS OF PROFESSIONAL CONDUCT

Sponsors and Executives must comply with the following Standards of Professional Conduct:

I. PROFESSIONALISM

A. **Knowledge of the Law.** Sponsors and Executives must know and comply with all applicable laws, rules, regulations and codes (including the Listings Requirements and the Code and Standards) of any government, regulatory organisation, licensing agency, or professional association governing their professional activities. In the event of a conflict of these laws and/or rules, regulations or codes, Sponsors and Executives must comply with the more onerous of the law, rule, regulation or code.

B. **Independence and Objectivity.** Sponsors and Executives must exercise reasonable care and judgment in order to achieve and maintain independence and objectivity in their professional dealings. Sponsors and Executives must not offer, solicit, or accept any gift, benefit, compensation or consideration that may reasonably be seen to compromise their independence or objectivity.

C. **Faithful Representation.** Sponsors and Executives must not knowingly make any misrepresentations or omissions of fact in relation to the provisions of the Listings Requirements. Sponsors and Executives must, without delay, inform the JSE .in the event that they become aware of any such misrepresentations or omissions of fact by, or on behalf of, their clients (whether existing, former or prospective).

D. **Misconduct.** Sponsors and Executives must not engage in any conduct involving dishonesty, fraud, deceit or the commission of any act that may reflect adversely on the JSE or on the professional reputation, integrity, or competence of the Sponsor or Executive.

II. INTEGRITY OF CAPITAL MARKETS

A. Material Non-public Information. Sponsors and Executives in possession of material price-sensitive, non-public information must not trade on or disclose this information to third parties (unless a legal obligation of disclosure exists).

B. False Markets. In order to protect the integrity of the capital markets, Sponsors and Executives must refrain from prohibited market practices and false statements, as defined in the Securities Services Act 36 of 2004, and take steps to make their clients aware of their responsibility in this regard.

III. DUTIES TO CLIENTS

A. Prudence and Care. Sponsors and Executives should act with reasonable care.

B. Fair Dealing. Sponsors and Executives must deal fairly and objectively with all clients when furnishing advice on the Listings Requirements or engaging in other professional practices relating to their duties as sponsors.

C. Preservation of Confidentiality. Sponsors and Executives must keep confidential all and any information pertaining to existing, former and prospective clients, unless:

1. The information relates to illegal activity on the part of the existing, former or prospective client;
2. Disclosure of the information is required by law or in terms of the Listings Requirements; or
3. The existing, former or prospective client consents to the disclosure of the information.

IV. CONFLICTS OF INTEREST

A. Disclosure of Conflict. Sponsors and Executives must make full and fair disclosure to both their clients and to the JSE of all matters that might reasonably be expected to impair their independence and objectivity or to conflict with their obligations to their clients or prospective clients. Where disclosure of any conflict of interest is included in shareholder documentation, Sponsors and Executives must ensure that such disclosure is presented prominently, is worded in plain language and that it communicates effectively the relevant information.

Reference: https://www.**jse**.co.za/

CHAPTER 3 : FINANCIAL MATHEMATICS

3.1 Introduction

A Rand that needs to be paid or received at different times will have different values. For example, a Rand today will be worth less than the same rand in two years. The question that needs to be answered is why will it be worth more in two years? It will be worth more as the Rand can be invested today for two years and it will earn a return *(interest)* resulting in a greater value in two years. The concept is known as time value of money *(TVM)* and the process is called compounding of interest i.e. the future value *(FV)* of one Rand is more than the present value *(PV)* on one Rand. The vice versa also applies, i.e. the present value *(PV)* today of one Rand is less than the future value *(FV)* of one Rand at any point in the future. As with any calculation of financial values, it is of importance to consider the mathematical process involved in the calculation of values.

3.2 Mathematics

As indicated in the introduction the future value of one Rand is greater than the present value of the same rand today. The future value of a Rand invested today to create a larger value in the future (*FV)* can be calculated by applying the following formula:

$$FV = PV(1+i)^N$$

Where: *FV* is the future value of a Rand amount, invested today; *PV* is the present value of a Rand today; *i* is the interest rate or the interest to be earned on the investment, written as a decimal, on the Rand amount invested; and *N* is the number of periods of the investment.

3.3 Percent

The word *percent* means *per one hundred* or divided by one hundred. For example, 45 percent (%) is equivalent to the fraction $\frac{45}{100}$ or to the decimal of *0.45*. The decimal is the result of the division of 45 by 100. Therefore to convert a percent, *from percent* to *decimal form*, take the number in front of the percent symbol, %, move its decimal two places to the left and drop the percent symbol.

Example: 8.5% = 0.085 and 0.5% = 0.005
It is important to note that the reverse also applies i.e. if the number is written in *decimal form* to change it into percentage form the decimal point must be moved two places to the right and the percent symbol, %, must be added.

Example: 0.83 = 83%
Note: Many mathematical formulas like the future value formula above needs the interest rate to be entered as a decimal, example 1, even though the interest is given as a percentage.

3.4 Order of mathematical operations

When performing any calculation, financial or mathematical, it is important to following the correct order of operation in simplifying and arithmetic expression:

- First, perform all *exponentiation*, proceeding from left to right i.e. powers and roots;
- Second, perform all *multiplication and divisions*, proceeding from left to right;
- Third, perform all *additions and subtractions*, proceeding from left to right;
  - Example: $3+4\times5 \Rightarrow 3+20=23$
  - Example: $7\times3^2 \Rightarrow 7\times9=63$
- Simply expressions in *parentheses first*, using rule 1, 2, and 3. If one set of parentheses is embedded in another set, work from the innermost set outwards and;
- Always work from left to right.

Note: If grouping symbols such as parentheses "( )", brackets "[ ]", and braces "{ }" are present the operations are simplified by first starting with the innermost grouping symbols and then working outwards. For example:

$$\{[12-2\times(7-2\times2)]\div3\}^2$$
$$\{[12-2\times(7-4)]\div3\}^2$$
$$\{[12-2\times3]\div3\}^2$$
$$\{[12-6]\div3\}^2$$
$$\{6\div3\}^2$$
$$2^2=4$$

3.5 Solving for *N*

Let's reconsider the FV formula:

$$FV=PV(1+i)^N$$

Where: *FV* is the future value of a Rand amount, invested today; *PV* is the present value of a Rand today; *i* is the interest rate or the interest to be earned on the investment, written as a decimal, on the rand amount invested; and *N* is the number of periods of the investment.

Consider the following example. David has deposited R1 000 *(PV)* into a savings account. He plans to withdraw his deposit as soon as it has grown to R2 000 *(FV).* Assume that he will be paid 4% interest compounded annually, how long will he have to wait before he will have R2 000?
The following timeline can be draw:

By substituting the known the variables in the *FV* formula we obtain the following:

$$FV=PV(1+i)^N$$
$$R2000=R1000(1.04)^N$$

It is clear from the above that the formula needs to be rearranged to say "N = …….." Unfortunately, there is no simple way to do the rearranging, without using logarithms. Logarithms in mathematics are used to simplify working with exponents or powers, *see section exponents.* There are several rules or laws of logarithms or "log" that define the applying of logarithms to equations and the change that will take place in an equation. In the above example, we need to "move" the exponent *N* down to the primary equation line and the try to rearrange the formula to be "*N* = …." We cannot divide both sides by *N* nor can we divide both sides by 1 000$(1.04)^N$. To resolve the problem we will use logarithms, but before using logarithms let us divide both sides with 1 000 first, we are left with:

$$\frac{2000}{1000}=\left(1+0.04\right)^N$$

Before solving for *N*, it is important to investigate the "laws of logarithms".

The "laws of logarithms."
Consider the following equation $8 = 2^N$, we can solve the equation by "taking" the log of both sides of the equation. This will result in the following equation: *log (8) = log* (2^N)*.* What this new equation means and what does it say? The problem we have is resolved by one of the "laws of logarithms". The "law" states that the log of a number to a power is equal to the power multiplied by the log of that number. By applying the "law", we can rearrange the equation to: *log (8) = N × log (2)*. Now we

can solve the equation by taking the log of 8 divided by the log of 2. The value of *N* for which 2 taken to the power is equal to 8, therefore, the value of 3.

If we take the log on both sides of the equation we can re-write the formula as follows:

$$\log\left(\frac{2000}{1000}\right) = N \times \log(1+0.04)$$

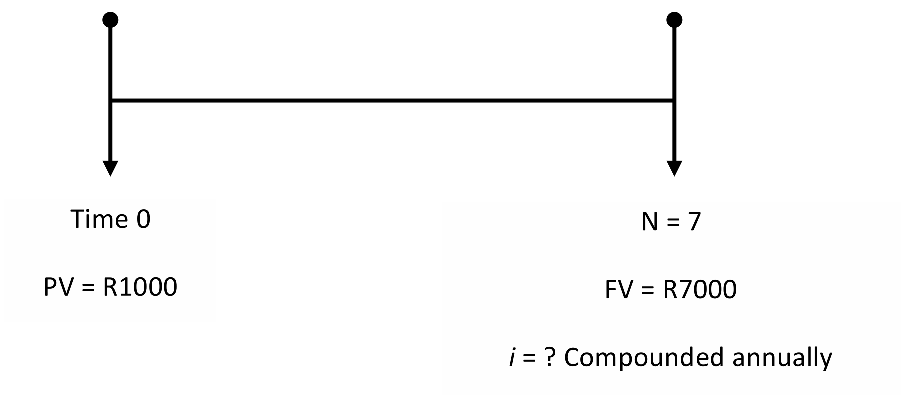
We can now solve for *N* by dividing both sides by *N*:

$$N = \frac{\log\left[\frac{2000}{1000}\right]}{\log(1+0.04)}$$

3.6 Solving for *i*

Problems requiring us to solve for *i* are not that prevalent, but it is still important to understand the math process required in solving for *i*.
Consider the following example: What interest rate, compounded annually, would result in an initial deposit of R1 000 to grow to R7 000 in 7 years?

The following timeline can be draw:



By substituting the known the variables in the FV formula we obtain the following:

$$R2000 = R1000 \times (1+i)^7$$

In order to solve the equation we need to rearrange the formula to look like this:

$$\frac{R2000}{R1000} = (1+i)^7$$

It is clear that we need a further mathematical intervention, as was the case when solving for *N*, to solve for *i.*

Finding roots
Consider the following equation 4 = X^2. The question to be asked is; how would we solve this? The answer is, by taking the square root of both sides. By taking the square root both sides we can re-write the equation as:

$$\sqrt{4} = \sqrt{x^2}$$

However, the square root of X^2 is just X. So we are only left with:

$$\sqrt{4} = x$$

The equation can now be solved by using a calculator, enter 4 into the calculator and press the square root button. The answer is 2.

In your example, we need the take the "seventh root" on both sides of the equation, or take both sides of the equation to the $\frac{1}{7}^{th}$ power (exponent). This result in the following changes in the formula:

$$\sqrt[7]{\frac{2000}{1000}} = \sqrt[7]{(1+i)^7}$$

This can further be reduced to:

$$\sqrt[7]{\frac{2000}{1000}} = (1+i)$$

Since the “seventh root” of something raised to the seventh power cancel each other out. The formula can now be re-written in such a way that we can show *“i* = …." In this format, we can solve for *i*:

$$i = \left(\sqrt[7]{\frac{2000}{1000}} - 1\right)$$

3.7 The basics of time value of money

The concept of time value of money or *TVM* can be defined as the value derived from the use of money over time as a result of investment and reinvestment today. *TVM* may refer to either present-value *(PV)* or future-value *(FV)* calculations. The present value is the value today of an amount that would exist in the future, given a stated rate of return or interest rate. Future value is the value in the future of a known amount today, given a stated rate of return or interest rate.

Before discussing the concept of the time value of money, further it is of importance to consider certain important issues. Firstly, the assumptions inherent in the time value of money calculations. There are several important assumptions underlying the TVM equations and it would not be practical to perform calculations without being fully aware of their influence:

- Money is always invested and always productive so that returns can be reinvested at a rate equal to *i*
- The yield curve is flat so that short-term interest rates are equivalent to long-term interest rates. In other words, there is no difference between long and short-term interest rates. This is almost never the case. In choosing the right interest rate, the time horizon exercises a powerful influence. At a minimum, the interest rate should always be adjusted for the time to maturity
- Time periods are all of equal length
- Payments are all equal and either all inflows or all outflows
- The interest rate is constant throughout the term
- Annuities are simple, certain, discrete and ordinary

The last assumption i.e. annuities are simple, certain, discrete and ordinary requires further explanation:

- Certain annuity: one with a fixed number of payments and the assurance that the payments will be made (i.e., they are not contingent on any event that cannot be entirely foretold)
- Discrete annuity: one with equal intervals between successive payment dates
- Simple annuity: one with payments and interest conversions on the same date
- Ordinary annuity - one in which all the payments are made at the end of the period

It is of importance to address the restrictions that are imposed on the calculation of TMV by the assumptions. Often you'll find in real-life that problems don't fit neatly into formulas and assumptions that have been derived from theory. Fortunately, there are ‘work-arounds” for many of the restrictions imposed by the assumptions above. A very versatile approach to overcoming the restrictions of overburdening assumptions is the notion that complex things are built from combinations of simpler things. In other words, break the problem up into smaller pieces, i.e. un-even payments, changing interest rates, and un-equal time periods?

All of these issues infringe the basic assumptions mentioned above. However, they can all be handled quite easily. Remember, any annuity can be broken up into a series of individual single sums. Likewise, a single sum with a large term can be broken down into a series of smaller single sums with shorter terms. In either case, the 'aggregate' present or future value is simply the summation of all the individual pieces.

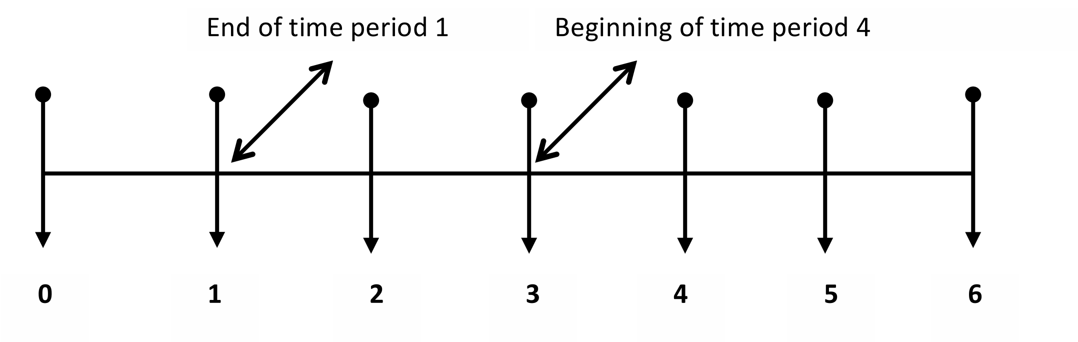
Other factors that may need to be considered are credit risk, inflation, taxes, options or unusual contractual terms, the nature and type of investment, alternative investments, and anticipated economic activity. The need for precision is also a significant factor to consider when deciding on an interest rate.

The terminology used in TVM calculations is not precise. For example, in practice the term interest rate, discount rate, yield, and rate of return are often used interchangeably. In an effort to avoid ambiguousness, here are some basic definitions that may be helpful:

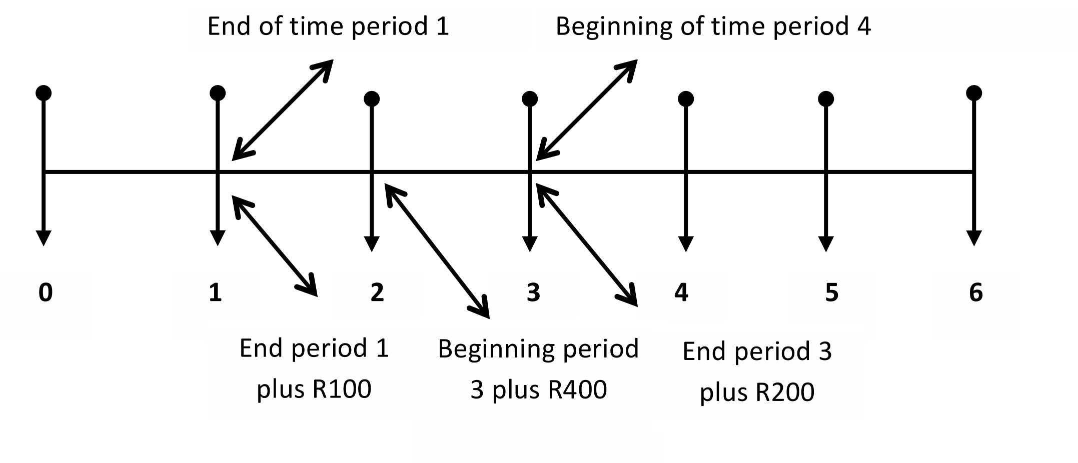
- Money that is loaned earns money for the lender and such money is called **interest**
- The amount of money which is owed and upon which the interest is earned is called **principal**
- The **rate of interest** in a given interval is numerically equal to the interest earned in one interval on a unit of principal per unit of time
- The **term** is the interval extending from beginning of the first compounding period to the end of the last compounding period
- **Compound interest** is interest charged on interest
- **Compounding frequency**
    - Annually (NACA): ***m = 1***
    - Semi-annually (NACS): ***m = 2***
    - Quarterly (NACQ): ***m = 4***
    - Monthly (NACM): ***m = 12***
    - Weekly (NASW): ***m = 52***
    - Daily (NACD): ***m = 365***
- The **nominal rate** of interest is the stated annual rate of interest not taking into account compounding
- The **effective rate** of interest is the actual annual rate of interest taking into account the effect of compounding
- An **annuity** is a series of periodic payments
- The **accumulated value** of an annuity is the total accumulated values of all payments and interest as of the end of the annuity term
- A **coupon bond** is a bond that makes periodic (usually semi-annual) interest payments
- A **discount bond (i.e. zero-coupon bond)** is a bond that does not make periodic interest payments. Instead, it is sold at a 'discount' and the difference between its face value and the price paid is the equivalent of a single interest payment made at maturity

3.8 The representation of time

What does it mean to say that you will receive R100 per year for five years? When will the money be paid at the beginning of each year, in the middle of each year or at the end of each year? A simple solution to the problem is to draw a graphic representation of the cash flows. The following timeline can be draw:

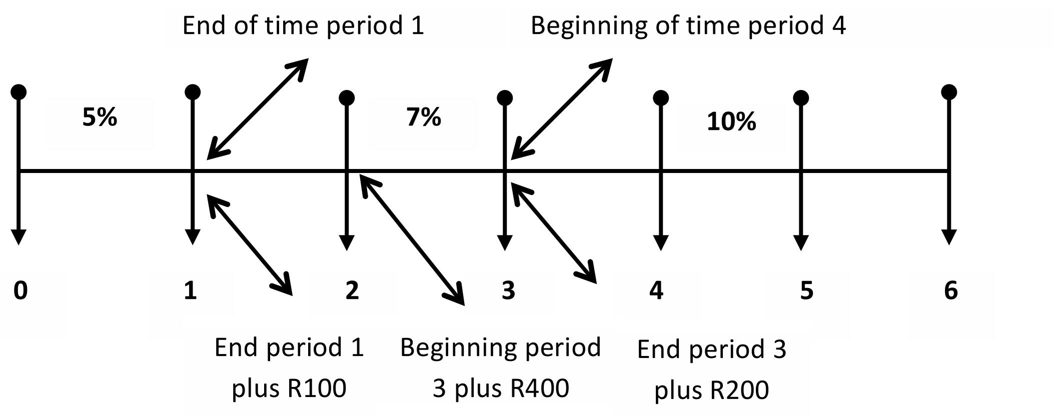


Time 0 (zero) is the present, the right here and now, but it is also the beginning of the first time period. *Time 1* is the end of time period one and the beginning of time period 2. *Time 3* is the end of time period 3 and the beginning of 4. The interval between the time periods on the timeline i.e. between 1 and 2 can be years, months, weeks, or even days. The cash flows are written on the timeline when they actually occur. Suppose that you will receive R100 at the end of period 1 and R400 at the beginning of period 3 and R200 at the end of period 3. These cash flows can be represented on a timeline as follows:



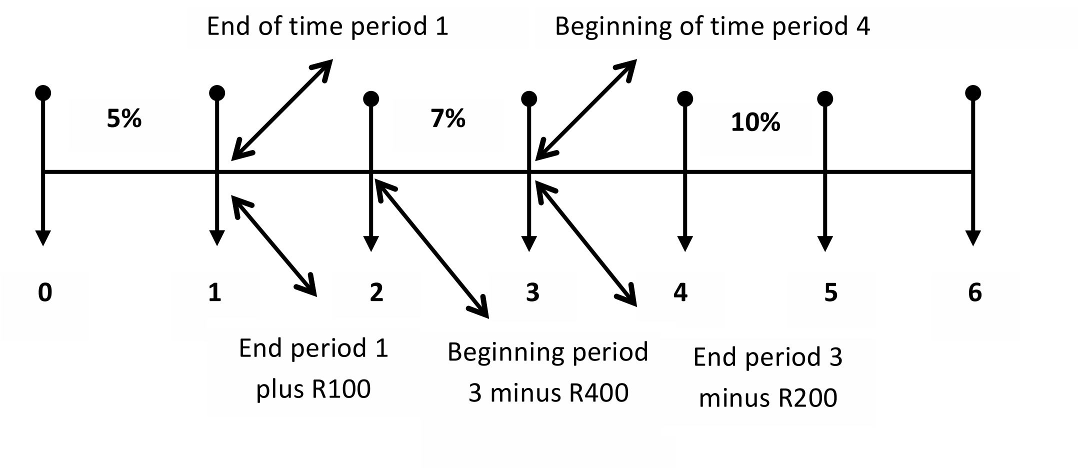
Note: Be particularly careful when putting the cash flows given in a word problem onto a timeline. If the problem says the cash flow occurs at the end of the time period, do not put it at the beginning of that time period

If an interest rate is specified, it is placed between the tick marks on the specific timeline. On the timeline below, 5% interest will be paid in time period 1 and 2, 7% in time period three and 10% in periods 4 to 6. The following timeline can be draw:



Note: Pay attention to the difference between points in time and periods of time. The numbers on our timeline represent points in time, but growth in the value of money occurs during the intervals between the points

By convention, if the interest rate is the same for consecutive periods we do not repeat the interest between each interval. It must be remembered that when calculating the time value of money cash flows can be positive or negative i.e. inflows, positive and outflows, negative. Inflows or cash received have a positive sign "+" or as indicated in the timeline below it has no sign. A cash outflow has a negative sign as indicated on the timeline below.



Timelines are useful as an aid to solving TVM problems. They can help keep you organised, especially when the cash flows become complex. Even experienced finance professors rely on them to help visualise problems.

3.9 Day count convention

The day count convention determines how interest accrues over time in a variety of transactions, including bonds, swaps, bills and loans. Interest is usually expressed to accrue at a rate per annum (the reference period). It is often due and payable at shorter intervals, usually a number of months (the interest period).

The day count convention regulates how the parties are to calculate the amount of interest payable at the end of each interest or other period. It is commonly expressed as a fraction. The numerator will be the convention for the number of days in the period - usually actual or a notional 30. The denominator is the convention for the number of days in the reference period - often 360 or 365.

Conventions vary, depending on the market type and location, and the currency in question. For example, euro-denominated bonds are usually calculated on an actual/actual basis while fixed rate non-euro denominated bonds are often calculated on a 30/360 basis. The London interbank market, on the other hand, operates on the basis of actual/360, except where the currency is Sterling, for which the London interbank convention is actual/365. Five basic day count convention excites:

1. Actual/360
2. Actual/365
3. Actual/Actual
4. 30/360

The day count convention is usually expressed as *X* ÷ *Y*. When calculating the interest earned between two dates, *X* defines the way in which the number of days between the two dates is calculated and Y defines the way in which the total number of days in the reference period is measured. The interest earned between the two dates can be calculated by using the following formula:

$$\frac{Number\ of\ days\ between\ dates}{Number\ of\ days\ in\ reference\ period}\ x\ Interest\ earned\ in\ reference\ period$$

3.10 Simple interest

Simple returns do not earn interest on reinvested interest. A balance is deposited and at the end of the investment period, interest is computed and paid. In the following example, we compute the simple interest earned on a deposit.

Example: Calculate the simple interest earned on R1 000 at 5% for one year:

$$Interest\ \ Earned = Amount\ \ Invested \times Interest\ \ Rate$$

$$Interest\ \ Earned = R1000 \times 0.05$$

$$Interest\ \ Earned = R50$$

If the investment was only for 90 days then the simple interest would be calculated as follows: Assume day count is actual/365:

$$Interest\ \ Earned = Amount\ \ Invested \times Interest\ \ Rate \times Period\ \ \left(Day\ \ count\right)$$

$$Interest\ \ Earned = R1000\left(0.05 \times \frac{90}{365}\right)$$

$$Interest\ \ Earned = R1000\left[0.0123287\right]$$

$$Interest\ \ Earned = R12.33$$

In the above calculation, we calculated the amount of (simple) interest that we earned on a 90-day investment. When we are calculating the total proceeds from the investment the formula will be:

$$Total\ \ Proceeds = Amount\ \ Invested \times \left(1 + Interest\ \ Rate \times \frac{Days}{Year}\right)$$

The only differences between the two formulas are that when we calculate the total proceeds' we add 1 to the *parentheses,* part between the brackets. If we apply the total proceeds formula to the same information in the example the answer will be:

$$Total\ \ Proceeds = Amount\ \ Invested \times \left(1 + Interest\ \ Rate \times \frac{Days}{Year}\right)$$

$$Total\ \ Proceeds = R1000\left[1 + \left(0.05 \times \frac{90}{365}\right)\right]$$

$$Total\ \ Proceeds = R1012.33$$

Simple interest is easy to compute and understand, but most of the time, we want our interest payments to be reinvested. For example, if you put money into a savings account, the bank automatically deposits the monthly or quarterly interest payments back into the account so that during subsequent periods interest is earned on a higher balance. This is an example of compound interest, which is interest earned on interest.

At this point, it is important to differentiate between a nominal interest and an effective interest. A nominal interest is the stated rate of interest, exclusive of any compounding, that is paid on an investment. Annual interest of R80 on an R1 000 investment is a nominal rate of 8% whether the interest is paid in R20 quarterly instalments, in R40 semi-annual instalments or in an R80 annual payment. Use of nominal rates can be misleading when comparing returns from different investments.

An effective rate is the rate of interest that incorporates compounding in the calculation used to determine the amount of interest to be earned on the investment. For amounts reinvested during an entire year, the annual effective rate of interest multiplied by the principal will equal the amount of earned interest. Virtually all of the calculations performed in finance require compound interest calculations.

3.11 The future value of a sum

The future value measures the nominal future sum of money that a given sum of money is "worth" at a specified time in the future assuming a certain interest rate, or more generally, rate of return; it is the present value multiplied by the accumulation function.

3.12 Annual compounding

Compound interest arises when interest is added to the principal investment amount, so that from that moment on, the interest that has been added also earns interest. This addition of interest to the principal is called compounding.

If you make an R100 deposit into a bank that pays 5% interest once per year, you will have R105 at the end of one year. We call the amount we have today the present value and the future balance the future value. In this case, we found the future value by multiplying the R100 by one plus the interest rate (1+*i*). Where did the "1" come from? The"1" adds the original balance back to the interest that has been earned to give the total balance in the account, see the total proceeds examples. The formula for computing the balance after one time period is:

$$FV_1 = PV_0\left(1+i\right)$$
$$FV_1 = R100\left(1+0.05\right)$$
$$FV_1 = R105$$

The process of computing a future balance is called compounding because the investor is earning compounded interest. Notice the use of the subscripts in the above formula. The subscript denotes the point on the timeline when the cash flow occurs.

Note the subscripts on the FV and PV variables. The subscripts refer to the time periods. PV_0 means the PV at the beginning of time period 0. FV_1 refers to the FV at the end of time period 1.

Now suppose that you leave the above deposit in the bank to compound for another year without withdrawing any money. Using the above formula, the balance grows to:

$$FV_2 = FV_0\left(1+i\right)$$
$$FV_2 = R105\left(1+0.05\right)$$
$$FV_2 = R110.25$$

During the first year or time period 1, the investment earned R5; during the second year it earned R5.25, which is R0.25 more than in the first period. The extra R0.25 was from the interest earned on the first period's interest. We can simplify these calculations by noting that FV_1 is equal to $PV_0\ (1+i)$. By substituting $PV_0\ (1+i)$ for FV_1 into the above equation, we get:

$$FV_2 = PV_0\left(1+i\right)\left(1+i\right)$$
$$FV_2 = PV_0\left(1+i\right)^2$$
$$FV_2 = R100\left(1+0.05\right)^2$$
$$FV_2 = R110.25$$

If we leave the funds for a third year we are left with the following:

$$FV_3 = FV_2\left(1+i\right)$$
$$FV_3 = R110.25\left(1+0.05\right)$$
$$FV_3 = R115.76$$

We can simplify the process once more, by substituting $PV_0\ (1+i)^2$ for FV_2, we get;

$$FV_3 = PV_0\left(1+i\right)\left(1+i\right)^2$$
$$FV_3 = PV_0\left(1+i\right)^3$$
$$FV_3 = R100\left(1.05\right)^3$$
$$FV_3 = R115.76$$

As can be seen, each subsequent period of compounding increases the exponent by one. The generalised equation for finding the future value of a deposit is given by:

$$FV_n = PV_0\left(1+i\right)^n$$

Where: FV_n *is* the future value of a deposit at the end of the *nth* period; PV_0 is the initial deposit made in time period 0; *i* is the interest rate or rate of return during each period; and *n* is the number of periods the deposit is allowed to compound.

3.13 Future value, beginning-of-period investment (deposit)

What will the value of an investment be if R1 500 is deposited and allowed to compound for 20 years at 8% with interest being paid annually? How much interest will be earned on the original deposit?

$$FV_{20} = PV_0\left(1+i\right)^n$$
$$FV_{20} = R1500\left(1+0.08\right)^{20}$$
$$FV_{20} = R1500\left(4.66\right)$$
$$FV_{20} = R6991.44$$

To calculate the interest earned one have to subtract the future value from the present value:

$$Interest\ \ Earned = FV - PV$$
$$Interest\ \ Earned = R6991.44 - R1500$$
$$Interest\ \ Earned = R5491.44$$

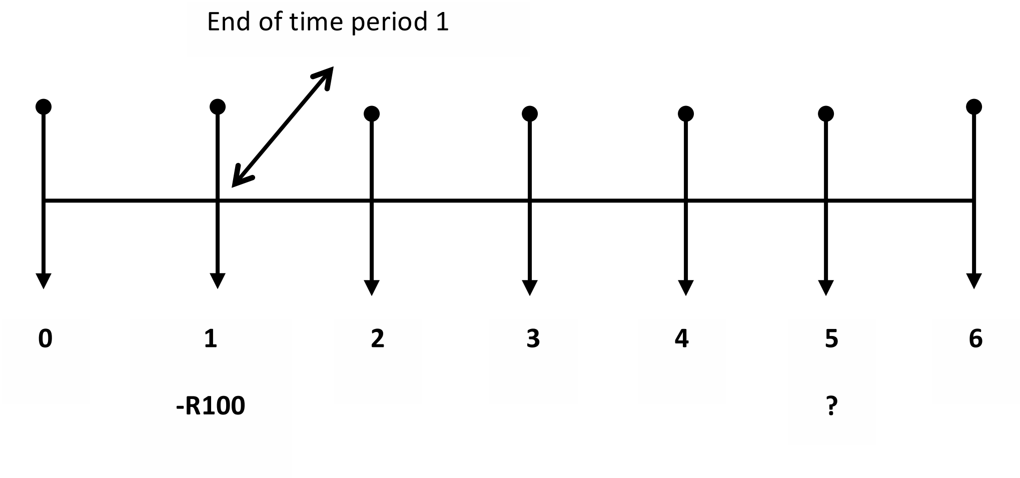
If simple interest, rather than compound interest, had been earned in the example above, R120 per year would have been earned (R1 500×0.08 = R120). Twenty times R120 is only R2 400. So R3 091.44 was earned because of compounding (R5 491.44 - R2 400 = R3 091.44). In other words, R3 091.44 of interest was earned on interest. In this example, more interest was earned on the interest than was earned on the original principal balance! This is the power of compounding.

Not all investments (deposits) are made at the present time. We sometimes need to compute future balances on deposits that will be made in the future. The method does not change. Simply count the intervals between when the investment (deposit) is made and the ending point on the timeline to determine the number of periods.

3.14 Future value end-of-period investment (deposit)

If a deposit of R100 is made at the end of the current period, what will be the balance in the account at the end of the fifth period if interest is paid annually at 5%?

To solve this problem, begin by drawing a timeline. Note that the initial R100 investment is given a negative sign because it is a cash outflow (money is going from your pocket into an investment):



To find the number of compounding periods *(n),* count the intervals during which the deposit can grow. There is one interval between 1 and 2, another between 2 and 3, a third between 3 and 4, and a fourth between periods 4 and 5, therefore there are four intervals, *n=4.*

$$FV_n = PV_1(1+i)^n$$

$$FV_4 = R100(1+0.05)^4$$

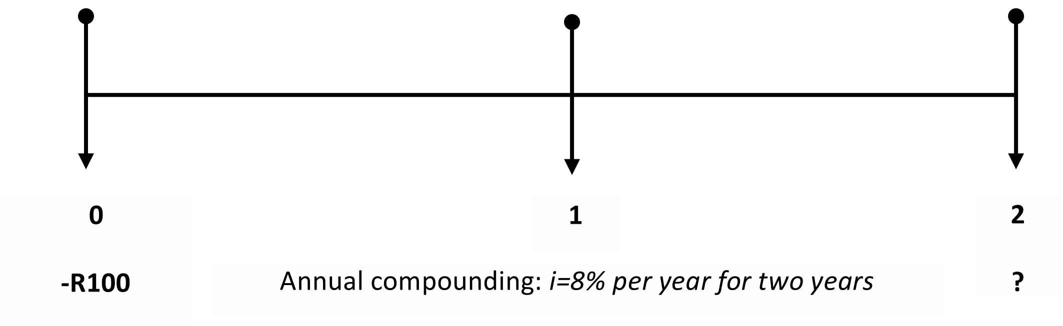
$$FV_4 = R121.55$$

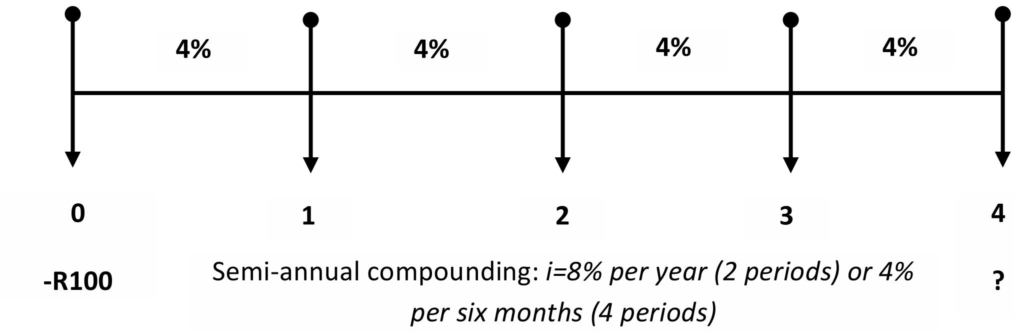
Note: Pay attention to the terminology used in the example above. The initial investment (deposit) was made at the end of the first period, yet we still called it the present value. The cash flow farthest to the left on the timeline is always the PV. The cash flow to the right is always the FV

3.15 Non-annual compounding periods

In the above examples, interest was paid once per year. This is called annual compounding. Often, interest is paid more often than once per year. We must adjust our formula to allow for any interest payment schedule that may arise.

Semi-annual compounding (twice a year) occurs when interest is paid every 6 months. To compute the future value under semi-annual compounding, recognise that the exponent in the formula for future value is the number of periods, not the number of years. Similarly, the interest rate is the interest paid during the period, not the annual interest rate. In the last section, when we were discussing annual compounding, each period was 1-year so *n* equalled the number of years and *i* was the annual interest rate. If we want to compute the future value when each period is 6-months long, we must adjust *n* and *i* to reflect the number of 6-month periods and the interest paid during each 6-month period. This is demonstrated by the following timelines:





Suppose that you deposit R100 for 2 years, and the bank pays interest semi-annually at an annual rate of 8%. There are four 6-month periods in 2 years, **so *n* must be multiplied by 2**. **If the interest rate is 8% for 1 year, then 4% is being paid every 6 months**. Substituting these figures into the FV equation, we have

$$FV_n = PV_0(1+i)^n$$
$$FV_2 = R100(1+0.04)^4$$
$$FV_2 = R116.99$$

Notice that the balance at the end of two years is R0.35 higher under semi-annual compounding than it would be with annual compounding [R100(1.08)^2 = R116.64]. To adjust to semi-annual compounding, we multiplied the number of years *(n)* by-the number of periods per year *(m)* and we divided the annual interest rate by the number-of periods per year. The equation below shows how to adjust for any number of compounding-periods per year:

$$FV_n = PV_n\left(1+\frac{i}{m}\right)^{mn}$$

Where: FV_n is the future value of a deposit at the end of the *nth* year; PV_n is the initial investment (deposit); *i* is the ***annual*** interest rate; *n* is the number of years the deposit is allowed to compound; and *m* is the number of times compounding occurs during the year.
Example: What is the future value of a R1 500 deposit after 20 years, with an annual interest rate of 8%, compounded quarterly?

$$FV_n = PV_0\left(1+\frac{i}{m}\right)^{mn}$$
$$FV_{20} = R1500\left(1+\frac{0.08}{20}\right)^{4\times 20}$$
$$FV_{20} = R7313.16$$

In the above example, we found that R1 500 deposited for 20 years at 8% compounded annually grew to R6 991.44. It turns out that the more frequently interest is paid; the greater is the future value. The increase in future value from additional compounding periods increases at a decreasing rate because the length of the compound periods is getting smaller.

Note: The formula for computing the future value, assuming continuous compounding, is $\boldsymbol{FV_n = PV \times e^{in}}$, *where* ***e = 2.71828, i*** *= annual interest rate and* ***n*** *= number of years.*

3.16 The effective rate

If the bank increases the number of compounding periods, the amount earned on a deposit increases. This implies that a higher effective interest rate (EIR) is being earned. The effective interest rate is the amount you would need to earn with annual compounding to be as well off as you are with multiple compounding periods per year. To determine the exact effective interest rate, given multiple compounding periods per year, we find the FV of R1 after 1 year and then subtract the initial Rand. What are left are the earnings for the year, which

equals the effective interest rate. The FV of R1 is found by applying $FV_n = PV_n\left(1+\frac{i}{m}\right)^{mn}$ with PV=1 and n=1.
The effective rate is computed by subtracting one (1). The effective interest rate can be computed by applying the following formula:

$$Effective\ Rate\ (EIR) = \left(1+\frac{i}{m}\right)^{m} - 1$$

Example: What is the effective rate of interest if 12% is compounded monthly?

$$Effective\ Rate\ (EIR) = \left(1+\frac{i}{m}\right)^{m} - 1$$

$$EIR = \left(1+\frac{0.12}{12}\right)^{12} - 1$$

$$EIR = 12.68\%$$

This result is interpreted to mean that an annually compounded interest rate of12.68% is equivalent to earning a 12% annual rate that is compounded 12 times per year. Investors are more interested in the effective rate of interest than they are in the annual rate. Suppose that you were attempting to choose between two bank savings accounts. The first pays 5% compounded annually and the second pays 4.9% compounded monthly. Which would you prefer? To answer this, you must compute the effective rate of each alternative and pick the largest one. The effective rate of the first is unchanged, 5%. The effective rate of the second is 5.01%, so you would choose the bank offering 4.9% compounded monthly.

3.17 Calculating a future balance

Another application of future value is computing the initial deposit required to accumulate a future balance. Suppose you want to have R5 000 in your bank account at the end of 10 years. If you can earn 5% annually, how much must you deposit today? To solve this problem, begin by writing down the formula for future value, then plugin the values that you know. For example,

$$FV_n = PV_0\left(1+i\right)^{n}$$

$$R5000 = PV\left(1+0.05\right)^{10}$$

$$\left(\frac{R5000}{\left(1.05\right)^{10}}\right) = PV$$

$$R3069.57 = PV$$

If you invest (deposit) R3 069.57 today in an investment that pays 5% annually, you will have a balance of R5 000 at the end of 10 years. However, if you invested in the stock market and earned 12% rather than 5%, your future balance would be R9 533.62 [R3 069.57(1.12)^{10}= R9 533.62] instead of R5 000.

3.18 Inflation adjustment

A similar application of future value is computing the change in purchasing power due to inflation.

Example: How much will you need in 20 years to have the same purchasing power that R100 has today if inflation averages 3% per year?

This example is a little different from the others in this section because there is no investment (deposit) and no future balance to calculate. On the other hand, the concept of future value applies. The R100 is compounded at 3% for 20 years. To find the equivalent future amount, the *FV* formula will be applied as follows:

$$FV_n = PV_1(1+i)^n$$
$$FV_{20} = R100(1.03)^{20}$$
$$FV_{20} = R180.61$$

If an item costs R100 today and inflation averages 3%, the item will cost approximately R180.61 in 20 years.

3.19 Solving for number of periods and interest rates

In all of the examples shown so far we have used the basic future value equation to solve for how much a deposit today can grow to in the future. There are occasions, however, when we already know both the future and present values and want to solve for one of the other variables in the equation.

For example, we may want to know how long it will take to accumulate a future balance given a known interest rate and initial deposit. We may want to solve for what average compounded rate of interest has been earned if we know how long a deposit has been invested and its current balance. Solving for these different variables is not difficult. Simply identify the future and present value amounts and at least one other variable and plug them into the following equation and use algebra, to solve for the unknown variable. In the next example, we solve for the interest rate.

Example: Just as you were about to enter college you learned that your great aunt Elizabeth had established a college trust fund for you when you were born 20 years ago. She deposited R5 000 initially. If the balance is now R19 348.42, what average compounded rate of return has been earned?

$$FV = PV(1+i)^n$$
$$R19348.42 = R5000(1+i)^{20}$$
$$3.86968 = (1+i)^{20} \quad (Divide\ both\ sides\ by\ R5000)$$
$$\sqrt[20]{3.86968} = (1+i)$$
$$i = \sqrt[20]{3.86968} - 1$$
$$i = 1.0700 - 1$$
$$i = 7\%$$

3.20 Insight behind compounding

It is important that the student of finance develop an understanding of how the compounding process works beyond simply applying the equations. Two things should be noted. First, as the number of compounding periods increases, the future balance increases. Second, as the interest rate increases, the future balance increases.

One of the more important features of TVM calculations is that the methods can be applied to anything that grows. We can also use the same equations to find future sales if sales grow at a constant rate. We can estimate future stock dividends as well if dividends are assumed to grow at a constant rate. The method can be applied to any constant growth situation, whether it is money, sales, profits, populations or trees.

3.21 Present value of a sum

Often we need to determine what the value is today of sums that will be received in the future. For example, if a firm is evaluating an investment that will generate future income, it must compare today's expenditures with expected future revenues. To compare sums across time, future values must be adjusted to what they are worth today.

Note: Go back and review the future value concepts if you have any doubts about your understanding so far. TVM concepts build on one another. You must master each level before moving onto the next.

Let's reconsider the FV formula:

$$FV_n = PV_0\left(1+i\right)^n$$

From the above formula, it is clear that the *PV* been defined in terms of present value. To find the equation for computing present value, we only need to rearrange the terms in the above equation. If both sides are divided by *(1+i)*^{n}, we get the equation for present value:

$$PV_0 = \frac{FV_n}{\left(1+i\right)^n} \quad or \quad PV_0 = FV_n\left[\frac{1}{\left(1+i\right)^n}\right]$$

The process of computing the present value of a future sum is called discounting. Remember that in real terms a future sum is not worth as much as a sum you have today, e.g. R100 is worth more today than in two years' time. To convert future amounts to their present values, the future amount must be reduced, or discounted i.e. the present value is always less than the future value.

Note: It is important to keep the terminology straight with regards to TVM. Compounding refers to the process of computing future values. Discounting refers to the process of computing present values. Further, the discount rate is the interest rate used when discounting.

Note: The formula for computing the present value, assuming continuous compounding, is:
PV_0= FV_n × e^{-in}, where e = 2.71828, i = annual interest rate, and n = number of years.

Example: In one year, David expects to receive R100 in repayment of a loan that he made to his sister, but the problem is that he needs the money today. How much would his dad give him today if he were to transfer the future payment due to him, by his sister, to his dad today? Assume that you have investment opportunities available at 10% that have similar risk.

The present value of the future amount is calculated by applying the following formula;

$$PV_0 = \frac{FV_1}{\left(1+i\right)}$$

$$PV_0 = \frac{R100}{\left(1.10\right)}$$

$$PV_0 = R90.91$$

David's dad would pay R90.91 for the right to receive R100 in one year. David's dad would pay this much because if he invested the R90.91 it would be worth R100 in one year:

R90.91 (1.10) = R100

The *PV* of R90.91 is therefore R100.00, discounted at 10%.

It is of importance to devote a great deal of time to learning present value techniques. One reason for this emphasis is that many investment decisions are made by converting all relevant cash flows into present value terms. In the next example, an investment opportunity is evaluated.

Example: Your great-grandmother has left you R100 000 in a trust fund that you cannot access for another 5years. You have decided that you really need this money now to pay your university fees. You discuss your problem with your attorney, who offers you R75 000 for an assignment of the proceeds of the trust. If you can get a student loan at 8%, should you accept your attorney's offer?

This problem can be solved two ways. You can find the future value of R75 000 and see whether it is more or less than R100 000 or you can find the present value of the R100 000 and see whether it is more or less than the R75 000. Using the present value method;

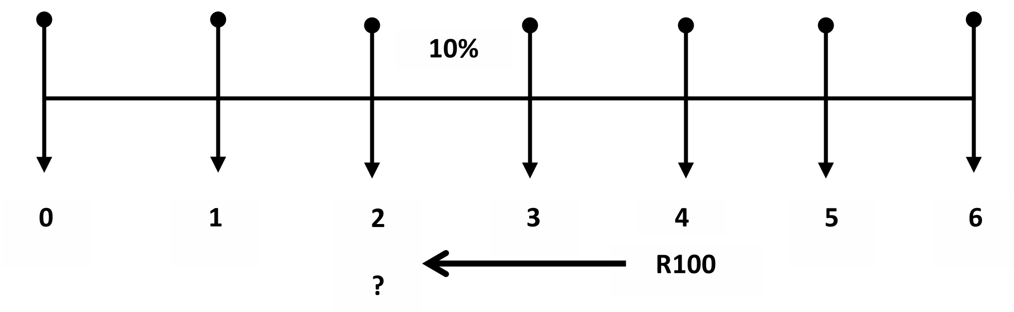
$$PV_0 = \frac{FV_1}{(1+i)}$$
$$PV_0 = \frac{R100000}{(1.08)^5}$$
$$PV_0 = R68059.62$$

Because the R100 000 trust payment is worth only R68 059.62 to you today, you would be happy to receive R75 000 for it today.

3.22 Present value timeline

You can think of discounting as moving Rand's to the left on the timeline. Whenever Rand is moved to the left, their value gets smaller. You do not always want to move cash flows all the way to time period zero. Present value calculations include moving future sums to any earlier period.

Example: Compute the value of R100 at the end of the second period. Assume that it will be received at the end of the fourth period and there is a 10% interest rate. The following timeline can be drawn;



There are two periods for the R100 to be discounted. PV_2 is the value at the end of the second period, which is computed as follows:

$$PV_2 = \frac{FV_1}{(1+i)}$$
$$PV_2 = \frac{R100}{(1.10)^2}$$
$$PV_2 = R82.64$$

3.23 Insight into present value

What does it mean that the present value of R100 to be received in 1 year at 10% is R90.91? It means that you are indifferent as to which you get: the R90.91 today or the R100 in 1 year. What if you do not really need the money today but expect to need it in 1 year? Then you can invest the R90.91 today and it will grow to be exactly R100 by the time you need it. The interest rate used to discount the R100 back to the present is selected so that you will be properly compensated for the risk that you may not be paid and for the delay in receiving the cash flow.

Still another way of defining present value is that if you have the present value of a future sum, you can exactly match that future sum by investing what you have today at the discount rate.

3.24 Present value of mixed streams

Thus far, we have focused on computing the present value of a single, lump sum future cash flow. However, there are many occasions when you must find the present value of a series of unequal cash flows.

For example, if you are evaluating an investment in a business, it is unlikely that each year's cash flows will be the same. To find the present value of mixed streams, simply find the present value of each cash flow individually, and then add them together.

3.25 Increasing the compounding periods

Computing present values with multiple compounding periods per year is very similar to computing future values with multiple compounding periods per year. The annual interest rate is divided by the number of periods per year to find the interest rate per period. The number of years is multiplied by the number of periods per year to get the number of periods the future value will be discounted.

3.26 Future value of an annuity

In the future value section, we computed the future balance of a single deposit or sum. Often, many equal deposits or payments are made. For example, you may make equal monthly deposits into a retirement account. You could find the future value of these deposits by computing the future value of each one separately and then adding them together, but this becomes tedious if there are many deposits. An easier way involves using annuities.

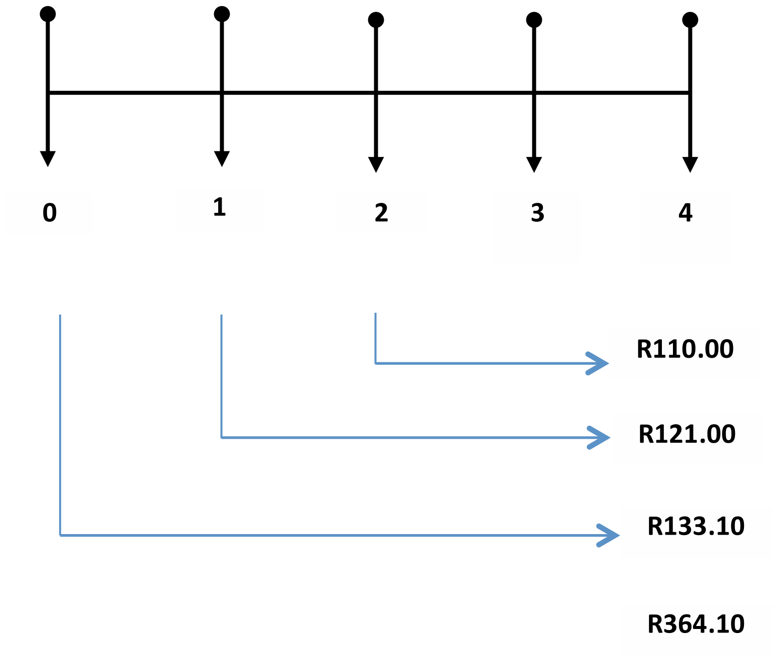
3.27 What is an annuity?

An annuity is a series of equal payments made at equal intervals. Despite being called annuities, annuity payments do not have to be made annually. They can be made monthly, weekly, or even daily. The critical factors are that the payments equal each other and that the interval between each one is the same.

An annuity in which payments are made at the end of each period is an ordinary annuity. For example, deposits to a retirement account typically are made at the end of each month when you get paid. An annuity in which payments are made at the beginning of each period is an annuity due. Apartment rental payments usually are due at the beginning of the month, so the annuity is called an annuity due. Ordinary annuities are more common than annuities due (as the term implies).

3.28 Computing the future value of an ordinary annuity

Suppose that you want to know the future balance in your account after 3 years if you make three equal annual deposits that each earns 10%. To solve this problem using $FV_n = PV_0\left(1+i\right)^n$ you would find the future value of each deposit, and then add them together.

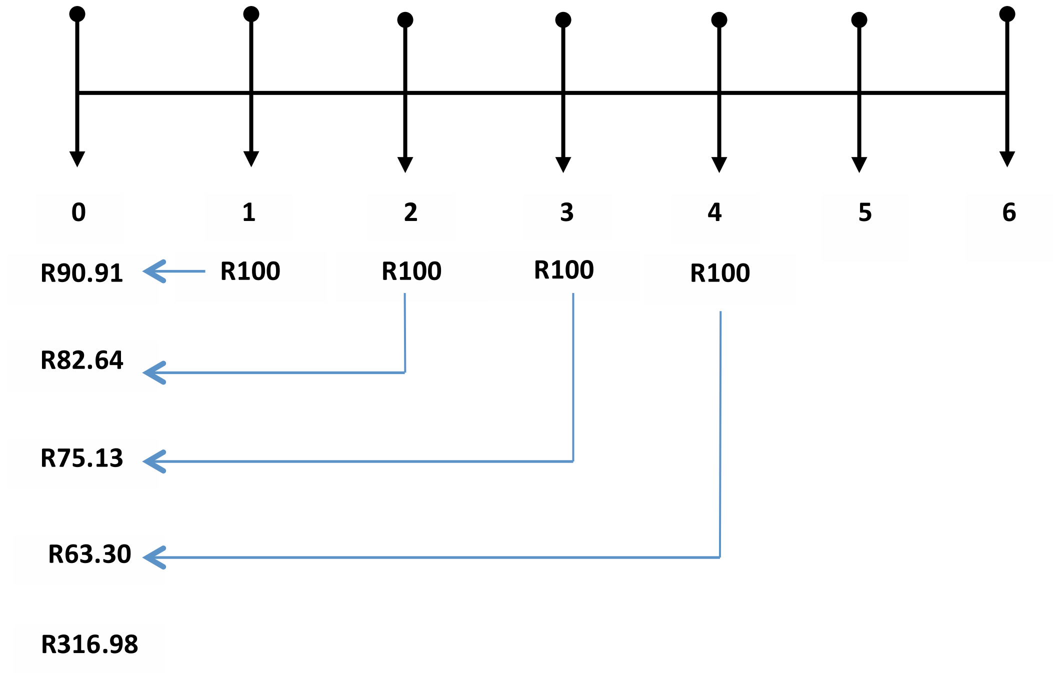


Notice that this is an ordinary annuity because the cash flows are received at the end of the period. Also, note that the last payment does not earn any interest. In other words, the last payment is deposited and the balance in the account is immediately checked. If the total future value is the sum of the future values of the individual payments, the following equation applies (where Future Value of Interest factor Annuity (FVIFA)):

$$FVA\left(FVIFA_{i,n}\right) = \frac{(1+i)^n - 1}{i}$$

3.29 Present value of an annuity

Just as we sometimes need to find the future value of a stream of equal cash flows, we also may need to find the present value of a stream of equal cash flows. If the cash flows are different from each other, there is no shortcut other than using a financial calculator. If the cash flows are equal to each other and occur at regular intervals, we can find the present value using the present value as follows. Review the following timeline:



One way to find the present value of the annuity shown on the timeline is to find the present value of each cash flow separately and then add them together, as shown. Obviously, if there are many cash flows, this can become tedious and time-consuming. The equation to solve the above problem could be written as (where Present Value of Interest factor Annuity (PVIFA)):

$$PVA\left(PVIFA\right)=1-\left[\frac{\dfrac{1}{\left(1+i\right)^{n}}}{i}\right]$$

3.30 Is it present value or future value?

Mastering the TVM techniques presented in this chapter is a significant step toward solving TVM problems. However, an added layer of complexity is the need to determine whether a particular problem is a present value or a future value problem. Consider the following example. You want to determine the amount that must be accumulated to pay a fixed R25 000 per year during your expected 20-year retirement, assuming a 10% interest rate.

Is this a present value, future value, present value of an annuity, or future value of an annuity problem? For most students, the hard part of learning TVM concepts is not mastering the equations, but rather determining which equation to use to solve a particular word problem

Remember, there were only four different equations presented in this document:

- Future value of a lump sum: $FV=PV(1+i)^{n}$
- Future value of an annuity: $FV=PMT(FVIFA_{i,n})$
- Present value of a lump sum: $PV=FV/(1+i)^{n}$
- Present value of an annuity: $PV=PMT(PVIFA_{i,n})$

3.31 Guidelines for TVM

- Use future value to find the balance resulting from an interest-earning deposit
- Use future value of an annuity to find the payment needed to achieve a known future balance
- Use either present or future value (without an annuity) to compute growth rates
- Use present value on all loan calculations
- Use present value to value assets
- Use present value to evaluate investments

To determine how to solve new, unfamiliar types of *TVM* problems and to pick which of the above equations to use, follow these steps: ***(1)*** Review the cash flows in the question to determine whether they are annuities. Once you have made this determination, only two equations remain. In the above example, because regular equal payments will be made, it is an annuity problem. ***(2)*** Review the information given in the problem and jot down what is known. If you are confused, draw a timeline. *PVs* are to the left and *FVs* are to the right. For instance, in the above example, we know that the regular payment amount is R25 000, so write down PMT = R25 000. We also know that *n = 20* and *i = 10%.* We do not know *PV* or *FV*. ***(3)*** Determine what you want to solve for. In the above example, we want to know how much is needed now to generate a series of payments *(PV=?).* This lets us choose an equation. We will use the *PV* of an annuity equation because it is the only one that includes all of the variables that are given plus the variable for which we need to solve. Here is one last suggestion. If you are attempting to solve a problem and find that you cannot determine what to plug into the equation for each of the variables, it may be because you are using the wrong equation. Try another.

Table: Time value of money formulae

Time value of money formula:	Annual Compounding	Compounding (m) Times per year	Continuous Compounding
Future value on a single or a lump sum	$FV_n = PV_0(1+i)^n$	$FV_n = PV_0\left(1+\frac{i}{m}\right)^{nm}$	$FV_n = PV_0 e^{in}$
Present value on a single or a lump sum	$PV_0 = \frac{FV_n}{(1+i)^n}$ or $PV_0 = FV_n\left[\frac{1}{(1+i)^n}\right]$ or $PV_0 = FV(1+i)^{-n}$	$PV_0 = \frac{FV_n}{(1+i)^{nm}}$ or $PV_0 = FV_n\left[\frac{1}{(1+i)^{nm}}\right]$ or $PV_0 = FV\left(1+\frac{i}{m}\right)^{-nm}$	$PV_0 = FVe^{-in}$
Effective interest rate given simple or quoted interest rate *(EIR)*	$EIR = i$	$EIR = \left[1+\frac{i}{m}\right]^m - 1$	$EIR = e^{i-1}$
Simple or quoted interest rate given as an effective rate	$i = EIR$	$i = m\left[(1+EIR)^{\frac{1}{m}} - 1\right]$	$i = \ln(1+EIR)$
The length of time required for a single cash flow to grow to a specified future amount at a given rate of interest.	$n = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+i)}$	$n = \frac{\ln\left(\frac{FV}{PV}\right)}{m\ln\left(1+\frac{i}{m}\right)}$	$n = \frac{1}{i}\ln\left(\frac{FV}{PV}\right)$
The simple or quoted rate of interest required for a single cash flow to grow to a specified future cash flow.	$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$	$i = \left[\left(\frac{FV}{PV}\right)^{\frac{1}{nm}} - 1\right]$	$i = \frac{1}{n}\ln\left(\frac{FV}{PV}\right)$

CHAPTER 4: FINANCIAL MARKETS AND INSTRUMENTS

4.1. Financial Systems and Financial Markets

A **financial system** is a **system** that enables investors with excess funds (**lenders**) and investors who need funds (**borrowers**) to exchange funds.

A **financial market** is a **market** that brings buyers and sellers together to trade in **financial** assets such as shares, bonds, commodities, derivatives and currencies/foreign exchange.

The participants in the **financial markets** are issuers of securities, buyers of securities, the financial intermediaries and the broker-dealers, fund managers, exchanges and regulators. The term financial market therefore encompasses the participants and their dealings in particular financial claims (debt and shares), groups of claims, and the manner in which their demands and requirements interact to set prices for such claims (interest rates and prices of shares).

The following terminology and concepts are used in **financial markets**:

- Money market
- Bond market (also called Debt market)
- Share market (also called Equity market)
- Foreign exchange market
- Derivatives markets
- Real Investments
- Instruments of Investment vehicles
- Primary and secondary markets
- Market form: exchange-traded and OTC markets

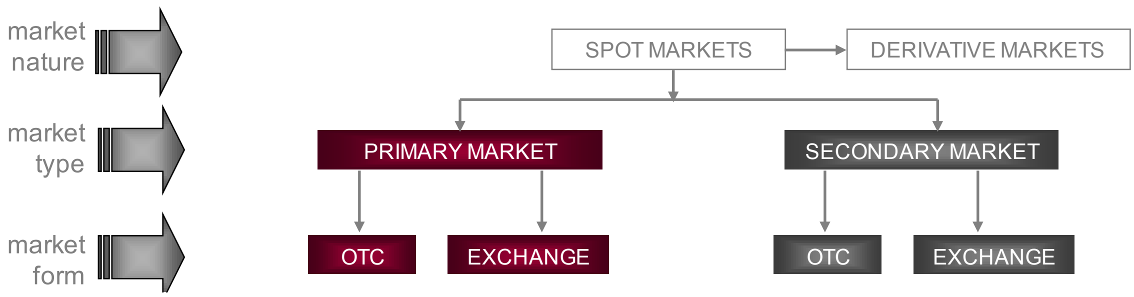
4.1.1. Financial Markets: Market Mechanism

The **market mechanism** is the structure, systems and conventions that exist to facilitate the issue and trading of shares. There are two types of market, i.e. the over-the-counter (OTC) market and the exchange-driven (and regulated) market. Most share markets around the world are exchange-driven markets.

In the case of the financial markets, however, all the markets start out as informal and some progress to formalised markets. For example, the forward markets are extremely useful markets and some have progressed into futures markets, not because the authorities want them so, but because the participants want them to be well-functioning, liquid and safe markets.

Some of the markets, such as the spot money market and the spot and forward foreign exchange markets never become formalised, and the reasons are straightforward: they work well as they are (i.e. without official intervention) and because they are the domain of intermediaries who themselves are sufficiently regulated.

Figure: Organisation of the spot financial markets including primary and secondary markets



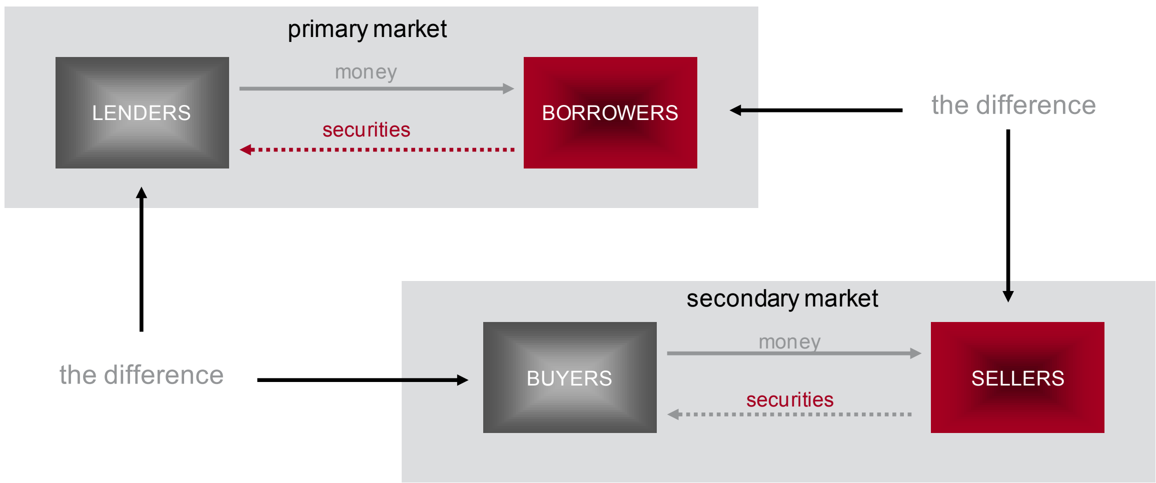
4.1.2. Financial Markets: Primary and Secondary Markets

4.1.2.1. Primary Markets vs Secondary Markets

Primary Market, also called the **New Issue Market**, is the market for issuing new securities (equity or bonds). The main players of these markets are the private and public companies that offer equity or debt based securities such as stocks and bonds in order to raise money for their operations such as business expansion or modernization. The primary market is a market for new capital that will be traded over a longer period. For example, if Telkom needed to raise money, Telkom, the company, would effectively sell shares to private investors.

Secondary Market is the market where investors trade among themselves, i.e., investors trade previously issued securities without the issuing companies' involvement. For example, if you want to buy Telkom shares, you would go to the JSE where you would be buying Telkom shares from another investor who owns Telkom shares. Telkom the company would not be involved in the transaction.

Figure: Primary and secondary markets



4.1.2.2. Economic Functions of Secondary Markets

Price discovery

Price discovery is one of the central functions of secondary markets. It is the route through which securities markets arrive at prices for the securities traded. Price discovery is important because it provides information that influences economic decisions, for example whether a company will expand production and finance this with long-term borrowing or the issue of new shares (rights offer). Price discovery also provides clues as to the prices that need to be offered on new issues of securities.

There are two prices in the various securities markets: *bid* and *offer* (or bid and *ask* in some countries). The *bid price* is the price that buyers are prepared to pay and the *offer price* the price at which holders of securities are prepared to sell. The bid price is always lower that the offer price (the opposite applies in the case of interest rates), and the difference between the two rates / prices is called the *spread*.

The *spread* is a valuable piece of information, for two main reasons. Firstly, it represents the cost of trading, i.e. it is a transaction cost, and this is significant in the creation or lack of liquidity (see below). Secondly, the spread is a reflection of marketability / liquidity. If the spread is narrow, the relevant market is said to be *liquid*, and if the spread is wide the market is *illiquid*.

Liquidity and borrowing cost reduction

Liquidity (some say *marketability* - these two terms mean the same in the financial markets) refers to the ability to trade a security with ease, i.e. without impacting significantly on its price.

Liquidity is significant for two main reasons. Firstly, it enables investors to rapidly adjust their portfolios in terms of size, risk, return, liquidity and maturity. This in turn has a major influence of the *liquidity premium* investors place on liquid securities. This, of course, means that the issuer is able to *borrow at a lower cost* than in the absence of liquidity. It is for this reason that many issuers of bonds attempt to create their own markets by acting as market makers (quoting buying and selling prices simultaneously) in their own securities, or by outsourcing this function to an investment / merchant bank/s. It is notable that central governments usually jumpstart the bond market by market making in their own securities (or outsourcing this to the central bank).

Support of primary market

The secondary market plays an important role in terms of *supporting the primary market*. We noted above that price discovery in the secondary market assists the primary markets in terms of providing clues as to the *pricing of new issues*. In addition to this important function, the secondary market provides clues as to the *receptiveness of the market for new issues* (which is reflected in the spread). Clearly, a liquid market improves the ability of issuers to place securities and lowers the price.

Implementation of monetary policy

An active secondary market enables the central bank to buy and sell securities in order to influence the liquidity of the banking system, with a view the ultimately influencing interest rates. This is termed open market operations, which means that the central bank buys and sells securities in the open market.

4.1.3. Financial Markets: Types

4.1.3.1. Money Market

The dominant players in the money market are the private sector banks, the central bank, the retirement funds and the money market unit trusts. The issuers of deposit securities, i.e. the central bank and the private sector banks. The central bank in most countries is the sole issuer of notes and coins. It may seem strange to call these deposit securities, but they are: the public and banks that hold these have made a deposit with the central bank.

The central bank is usually the sole banker to the government, and these deposits are not negotiable; hence the term NNCD applies here (non-negotiable certificates of deposit). The central bank also takes deposits from other sources; this differs from country to country but usually includes the foreign sector and certain other government entities; the term NNCD applies here also.

Many central banks also issue their own securities (central bank securities – CBS) and many terms apply here: Reserve Bank of Malawi bills, South African Reserve Bank debentures, Bank of Botswana certificates, Bank of Ghana bills, and so on. These are short-term securities and, therefore, fall into the money market. They are issued solely for monetary policy implementation purposes as they are similar to term deposits, and for the sake of simplicity, we refer to these CBS as deposits (negotiable certificates of deposits - NCDs).

The private banking sector issues two types of deposit certificates (securities): NNCDs and NCDs. The majority are NNCDs and most are of short term duration, making them money market instruments. NNCDs come in various forms, such as call deposits, fixed deposits, notice deposits, savings deposits and so on. NNCDs usually pay interest periodically, while NCDs are usually of the interest add-on form, i.e. an amount is deposited and interest is added to give a maturity value, payable at the end of the deposit period. NNCDs make up the vast majority of banks' deposits.

4.1.3.2. Bond Market

Formally, we define the bond market as:
The bond market is the mechanism / conventions that exist for the issue of, investing in, and the trading of instruments that represent the long-term undertakings (usually of a fixed capital nature) of the issuers.

Long-term bonds are issued to fund long-term undertakings, such as roads, energy and water delivery infrastructure and factories. Not all long-term borrowers are able to issue bonds. It is the domain (mainly) of government and the rates on government bonds are regarded as risk-free (actually *credit risk-free* because market risk is intrinsic to bonds). The rates on all non-government (i.e. corporate) bonds are benchmarked on the government bond rates.

It is only the large companies that are able to issue bonds, and their bonds are required to be rated by one or more rating agencies before any investor will consider them. In addition to these bonds, there are also bonds that are the products of securitisation, such as the bonds of non-bank mortgage lenders and others such as Collateralised Debt Obligations (CDOs). As these bonds are issued by Special Purpose Vehicles (SPVs) created for this purpose, we refer to them as SPV bonds. They also need to be rated before investors will consider them.

Products of securitisation are financial securities (products) of an SPV (securitisation vehicle) set up to hold assets that have a cash flow. The assets are financial (such as mortgages), or non-financial (such revenue from the sale of music) and they are financed by the securities issued.

As a result of the processes of borrowing, lending and financial intermediation, there is a wide range of *debt instruments* (evidences of debt / debt claims) in the financial systems of the world. A *debt instrument* can be defined as a claim against a person or company or institution (such as a government entity) for the payment of a future sum of money (the nominal / face / redemption value) and/or a periodic payment of money. In many instances there is no periodic payment of money (as in the case of treasury bills), while in others there is (as in the case of most long-dated bonds, interest on which is payable six-monthly in arrears). Similarly, there may be no promise of a sum of money in the future but a periodic payment only, as in the case of an undated bond (perpetual bond).

The debt market is made up of:

- The short-term debt market (STDM - claims up to a year, but this is an arbitrary term), which (when the deposit market is added) is referred to as the money market. This is our definition of the money market; some scholars prefer to only include marketable STDM instruments in this definition. There are good reasons for the wide definition, the main one being that price discovery largely takes place in the non-marketable part of this STDM.
- Long-term debt market (LTDM - claims from one year onwards). The bond market is the marketable arm of the LTDM, and claims in this market range from a year to deep into the future, in some cases up to 30 years. This does not apply in the non-marketable LTDM.

The most common money market instrument requires the issuer to pay a single amount at maturity, while the most common bond instrument requires the issuer to pay periodic interest and to redeem the claim on the maturity (due) date.

One of the most important characteristics of financial claims is that of *reversibility or marketability*. This refers to the ease with which the holders of securities can recover their investments, and can be achieved in one of two ways, i.e. by recourse to the issuer or by recourse to a secondary market (in which the holder can sell the claim).

4.1.3.3. Share / Equity Market

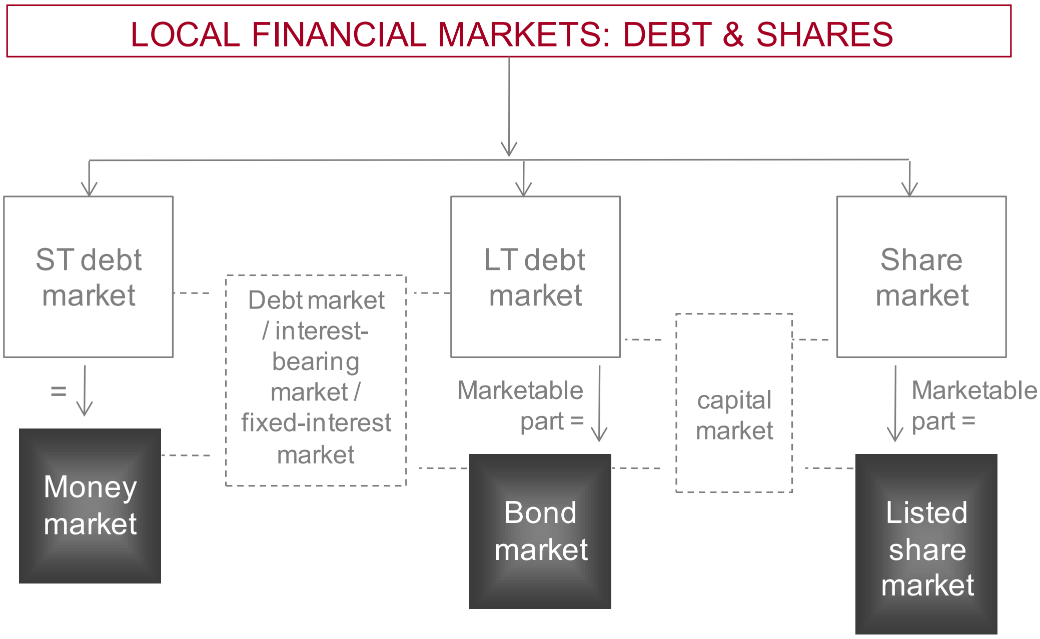
We define the share market as:
The share market is the mechanism / conventions that exist for the issue of, investing in, and the trading of marketable share instruments that represent the permanent or semi-permanent capital of the issuers (companies).

We also use the term "borrowers" for the issuers of shares because shares include preference shares which in many markets are redeemable. An ordinary share represents part-ownership and not a debt of a company. Shares are issued by companies in terms of the statute that regulates them (usually called the Companies Act) and there are two types:

- *Ordinary shares* (also called common shares or common stock) that represent the permanent capital of companies; they have no maturity date (as such they are much like perpetual bonds). These shares impart to the holder the right to vote on issues that affect the company. However, the shareholder does not have a right to the profits until the board of directors declares a dividend.
- *Preference shares* (also called preferred shares or preferred stock). These shares may be redeemable (i.e. have a fixed maturity date), redeemable at the option of the issuer or non-redeemable (have no maturity date). These shares impart to the holder the prior right over ordinary shareholders to the distribution of dividends and capital in the event of the company winding up.

Shares pay dividends, as opposed to bonds and money market instruments that pay interest. Dividends on preference shares are usually fixed-rate dividends and they have a preference over dividends on ordinary shares.

Figure: Local financial markets



4.1.3.4. Foreign Exchange Market

There is a market that is closely related to the money market: the foreign exchange market (from here on *foreign exchange* is termed *forex*). This market, strictly speaking, is not a financial market, because lending and/or borrowing does not take place in this market. The forex market merely makes it possible to *buy and/or sell forex*.

It is important to understand *what is meant by forex* and *buying and selling forex*. Forex means:

- Foreign notes and coins
- Foreign deposits and investments

Foreign notes and coins are bought for the purpose of spending in a foreign country. However, this is a tiny part of the forex market. The forex market is comprised mainly of the buying and selling and *foreign deposits*.

There are two types of transactions in the forex market: *spot* and *forward* (called forwards) transactions. Spot means *pay now*, which has a different meaning in different countries, and *forward* means at a date in the future other than spot. The forward market is a derivative market (from the spot market). In addition, these are other forex derivative markets, such as futures, options, swaps, etc.

The above should make it clear that the forex market is not a financial market in the sense that borrowing and lending take place. Rather it is a *conduit* for:

- Foreigners into the domestic goods and services markets (domestic notes and coin and travellers' cheques), to the domestic deposit market (receipts / payments for imports / exports), and to domestic financial markets (debt and share markets) (via the deposit market)
- Residents into the foreign goods and services markets (foreign notes and coin and travellers cheques), to the foreign deposit market (receipts / payments for imports / exports), and foreign financial markets (debt and share markets) (via the deposit market)

However, even though the forex market is not a financial market in the sense that borrowing and lending take place, it is a conduit to these markets, and the derivatives of this market are financial. Thus, we will give it the status of a financial market.

4.1.3.5. Derivative Market

In addition to the above-mentioned markets, there are a number of other related financial instruments that are called *derivatives*. The name arises from the fact that these instruments are *derived* from debt and share instruments (and other instruments), which mean that they cannot exist on their own, and they derive their value from the underlying debt, share and other instruments.

It must be added that there are also other derivative instruments that are derived not from debt instruments but from commodities (soft, such as grain, and hard, such as metals). In addition there are derivatives that are not derived from debt, shares or from commodities, such as weather derivatives; they are also financial instruments. Major types of derivatives are: forwards, futures, swaps, options, explained below.

Essentially, forwards and futures are contracts to buy or sell an asset (commodity, financial instrument or index) on a specified date in the future at a price determined upfront. An option is the same, except that the buy or sell is optional and the date is on the contract expiry date or before. Swaps are contracts to exchange cash flows on specified dates in the future, based on a notional amount.

We mention *spot* and *forward* markets earlier, and now need to explain these suitably. When a financial instrument is traded and settled on the same or on the following day or even five days hence, it is termed a *spot* transaction. This is usually written as T+0 or T+1 (in the case of most money markets), T+3 (in the case of most bond markets), and T+5 (in the case in most share markets). "T" denotes *transaction* date (or deal date), and the number after the “T” denotes the number of days after the transaction date when the transaction is *settled*. *Settled* means the relevant security is delivered and the consideration (the amount owing) paid for (this is termed *payment versus delivery*). All the instruments mentioned above are traded *spot*.

The spot settlement dates (T+number-of-days mentioned above) are the administratively convenient settlement days for the relevant markets. If, however, the settlement dates are on days other than the spot dates, they are *forward / future dates*. Thus, if a share deal is traded today for settlement in, say, two weeks, it is a *forward* transaction. The price of the forward transaction will be the spot price plus the price of money for a two-week term. A *forward* is thus *derived* from the spot market. Forward markets are derivative markets.

We now briefly describe the derivative instruments.

Forwards were mentioned earlier as the sale or purchase of a financial instrument on a date in the future, i.e. on a date other than the spot market date. The price of a forward is equal to the spot price plus the price of money (the interest rate) for the forward period.

Futures contracts are agreements to buy from, or sell to, an exchange established for this purpose, a standard quantity and quality of an asset (i.e. a financial asset, commodity or notional asset - like an index) on a specific date, at a price determined at the time of negotiation of the contract. Thus, the holders are obligated to perform.

Options (on “physicals”, i.e. on the actual spot instrument), on the other hand, give the holder the right, but not the obligation, to buy (call option) or sell (put option) the underlying asset at a predetermined price (strike price) during (an *American* option) or at the expiry (a *European* option) of a specified period. It will be evident that the option holders will exercise their options only if it is profitable to do so. Their potential profit is not fixed while their potential loss is limited to the amount of the premium paid. The writer has the opposite profile.

Options on forwards are options as described above, but the underlying instrument in a forward contract. Similarly, with options on futures the underlying instrument is a futures contract as opposed to a “physical”, i.e. a spot market instrument.

Swaps are contracts in terms of which certain payment obligations are swapped between two parties. With an interest rate swap (IRS), for example, a floating rate obligation is swapped for a fixed-rate obligation. The payments are based on a mutually agreed notional amount (amount the contract is agreed upon) that is not exchanged between the parties.

As noted there are also “other derivatives” and we have identified three: credit derivatives, weather derivatives and products of securitisation.

Credit derivatives are bilateral (two-sided) contracts between a protection purchaser and a protection seller that compensate the purchaser upon the occurrence of a *credit event* during the life of the contract. The *credit event* is objective and observable, and examples are default (or failure to pay), bankruptcy, rating downgrade.

Weather derivatives are hedges (risk management tool) against weather events. They rely on instruments such as caps, floors, collars, swaps, etc. and are settled in the same way as these. The counterparties to the hedgers use data supplied by independent organisations, such as the weather service data stations located at major airports.

4.1.3.6. Real Investments

Real investments are usually categorised into:

- Property
- Commodities
- Other (art, antiques, rare coins, rare stamps, etc.)

There are, of course, many subcategories to be found under each (see below). Real investments have many characteristics that differentiate them from financial assets such as:

- Zero recurring return (with the exception of commercial property)
- Inflation hedge
- Inefficient (illiquid) markets
- High transactions costs
- Insurance and storage (in the case of commodities and "other")
- High price volatility

Tangibility and pleasure (art, rare books, antiques)

4.1.3.7. Property

Of the real investments, property is the most significant investment for the retail investor (individual), and it usually makes up a large percentage of the portfolio (when young - because the individual typically owns this asset). However, in the case of wholesale investors such as retirement funds, property makes up a small proportion of assets (in most countries around 3-5%).

There are many forms of property investment:

- Undeveloped land (zoned residential, industrial, office, etc.)
- Developed farm (fruit, cattle, game, etc.)
- Residential (home)
- Multi-residential (block of flats)
- Retail (shopping centre, sectional title retail outlet)
- Office building
- An office (sectional title)
- Industrial building
- Leisure and tourism (hotel, resort, golf course, theme park)

Undeveloped land is purchased either to:

- Benefit from a price appreciation that is higher than the risk-free rate (i.e. the minimum guaranteed return) after taxes that may be levied on the property transaction (e.g. capital gains tax)
- Improve the property with the purpose of selling the improved property for a capital gain that is higher (the statement above applies here also). An example is the building of a block of flats with the purpose of selling them under sectional title. (In fact often the developer will only start building once a certain number of flats have been sold – to lessen the risk)
- Improve the property with the purpose of deriving a recurring rental income into the future. Examples are the building of a block of flats and the building of a shopping centre

With the exception of a residential home, the rest of the forms of property investment are held with the objective of rental income (or income in the case of a farm) in the main. Capital gain is usually a secondary motive unless the economic environment is one of high inflation. Then, capital gain becomes the primary objective of an investment.

The valuation of income-generating property is related to interest rates, i.e. the income on interest-bearing assets, the domain of the financial system's money and bond markets.

4.1.3.8. Commodities

As we have seen, "commodities" is the term for real assets / investments such as precious metals, grain, base metals, etc. These assets produce no recurring income and are invested in for capital gains only. There are many ways in which to categorise commodities, such as[1]:

- Hard commodities (non-perishable products or non-consumables)
    - Metals
        - Precious metals
            - Gold
            - Platinum
            - Palladium
            - Silver
        - Non-precious metals
            - Base metals (e.g. copper and iron)
            - Ferrous metals (e.g. steel)
            - Alloys (e.g. brass)
    - Minerals
        - Phosphates
        - Coal
        - Oil
        - Gas
- Soft commodities (perishable products or consumables)
    - Agricultural products
        - Crops
            - Vegetables
            - Fruits
            - Grain
            - Oilseeds
        - Livestock
            - Grazing
            - Poultry
            - Pigs
        - Products from livestock
            - Wool
            - Leather
            - Meat
    - Fishing products
        - Fish
        - Crustaceans.

Generally speaking investment portfolios do not contain a large proportion of commodities. The reason, as noted, is that commodities do not produce an income. Also, it is rare that *commodity* portfolios contain consumable products. Where investment portfolios contain commodities, the commodities are usually of the precious metal variety, particularly gold, platinum, silver and so on.

Precious metal investments take on many forms such as bullion, but the norm is coins, because of the convenience (compared with bullion). As noted, commodities do not yield a recurring return, only capital gains. Precious metals are also notably volatile at times; gold, for example, is a popular investment in time of unrest and uncertainty.

Often, investment in commodities takes the form of investment in investment vehicles, such as securities unit trusts (SUTs) and exchange traded funds (ETFs) (to be discussed later), mining shares and so on.

---

[1] The classification is from Faure, AP, 2005. **The commodity derivative markets**. Cape Town: Quoin Institute. It represents a personal view.

4.1.3.9. Other Real Investments

“Other real investments”, as we have briefly seen, include investments in items such as:

- Art of masters (such as Rembrandt)
- Antique furniture
- Rare stamps
- Rare books
- Rare coins

Generally speaking, investments in these items and in certain commodities (such as gold and diamonds), are not undertaken by the large investors such as retirement funds, but by high net worth individuals and reflect motivations such as:

- The desire for diversification of personal investments
- Personal satisfaction (aesthetic value)
- Survival (as in times of war)
- Inflation hedging

To this category, one can add other investments that do not have an aesthetic value, such as "tank containers". These investments have currency hedging and tax advantages.

Generally, investors expect capital gains from all real assets and a return only on certain non-undeveloped properties in the form of regular rental income. Many individual investors regard their residential property as their sole investment in real assets because it generally makes up a large proportion of their assets.

4.1.3.10. Instruments of Investments

The securities issued by the three investment vehicle categories (which we call participation interests – PIs) are highlighted, as well as the household sector (as the main lenders) which is our interest here. The institutions under each category are:

- Contractual intermediaries (CIs):
  - Long-term insurers (LTIs)
  - Retirement funds (RFs)
- Collective investment schemes (CISs):
  - Securities unit trusts (SUTs)
  - Property unit trusts (PUTs)
  - Exchange-traded funds (ETFs)
- Alternative investments (AIs):
  - Private equity funds (PEFs)
  - Hedge funds (HFs)

The instruments of the investment vehicles fall into a separate category because they are fundamentally different to those outlined above, and because they do not fit into the debt, share or deposit markets. All of them are also non-negotiable (with some exceptions). They invest in the instruments covered above (debt, shares, deposits and others such as property) and issue investment-type instruments suited to the specified needs of investors.

Long-term Insurers

In most countries, the statute covering life companies (long-term insurers / assurers) makes provision for the following different *classes of life business*. The insurers are obliged to register under one or more of these classes:

- Assistance
- Disability
- Fund
- Health
- Life
- Sinking fund

The products of these classes are called *policies*, for example, assistance policies, life policies, and so on.

Retirement Funds

Retirement funds (RFs) are the best-known investment vehicles. By retirement, most individuals' share of the fund of which they are a member (called member's interest, undivided share, participation interest) represents their largest asset; usually the next largest asset in terms of value is their residential property.

Retirement funds are *contractual savings institutions*, and they are similar to savings plans. Persons employed (the participants or members) and/or their employers contribute a certain amount of funds per time period (usually monthly) to the fund. This usually takes place during the working lifetime of the members, the *purpose being to provide financially for retirement.*

There are three types of retirement fund:

- Pension fund (also called defined benefit fund - rules of the fund provide a specified benefit at retirement)
- Provident fund (also called defined contribution fund - rules of the fund do not commit the fund to a particular benefit; the company and the employee contribute a specified amount to the fund)
- Preservation fund (a "parking" fund until retirement required by statute when a retirement fund participant leaves employment)

Securities Unit Trusts

The most popular investment vehicle for individuals is the securities unit trust (SUT). The SUT issues PIs (units) (say 100 000) to investors at a price (say LCC 100 per unit) and with the funds purchases the listed shares, bank deposits etc. to the value of the total funds available (LCC 10 000 000). If the value of the shares, etc. increases to LCC 12 000 000 (which is easily measured because the instruments are listed) a month later, each PI (unit) is now worth LCC 120 (minus the costs of managing the SUT).

There is a wide variety of SUTs; some examples are:

- Asset allocation targeted absolute return funds
- Equity value funds
- Equity financial and industrial funds
- Equity industrial funds
- Fixed interest bond funds
- Foreign equity general funds
- Real estate
- Worldwide equity technology funds

Property Unit Trusts

Property unit trusts (PUTs) are similar to SUTs in every respect except (mainly) in the nature of the asset portfolio (property) and the fact that they are listed. The purpose of a PUT is to provide smaller investors easy (i.e. small-amount) access to the property investment market, diversity in the property investment market, and professional management of the portfolio.

Exchange-Traded Funds (ETFs)

An exchange traded fund (ETF), also called a *tracker fund*, is a fund set up to track a particular index. It is a type of investment company whose investment objective is to achieve the same return as a particular market index. It invests in the securities of companies / government / commodities that are included in a particular market index. This means that the fund has liabilities in the form of PIs (also called shares and securities) which are listed on an exchange, and assets in the form of the specific shares / fixed-interest securities / commodities that make up the relevant index according to their weightings in the index.

An investment in a share index ETF is an inexpensive way of gaining exposure to a relevant segment of the share market, i.e. exposure is gained without having to purchase the individual shares that make up the index. Dividends are also payable to the holders of the shares of the ETF.

Private Equity Funds

Private equity fund (PEF) means a pool of funds that is available for investment in or are already invested in unlisted companies. The motivation for the formation of PEFs is usually to provide funding for entrepreneurial-type businesses that are highly regarded and to profit from the listing of these unlisted companies at some stage in the future. This institution is mentioned here for the sake of completeness. Individuals rarely invest in PEFs.

Private equity has become a separate asset class (some would say “becoming”), and in most countries where private equity funds exist so do industry associations. Private equity is associated with venture capital in that venture capital is seen as a form of private equity (start-up capital for the smaller companies). In most (if not all) cases the industry associations include this term. For example, the South African industry association is called the South African Venture Capital and Private Equity Association (SAVCA), the European one is named European Private Equity and Venture Capital Association (EVCA), the Italian one is called Italian Private Equity and Venture Capital Association (AIFI), and so on.

It should be evident that private equity funds are akin to investment companies on the liability side of their balance sheets, whereas their assets are comprised of investments in non-listed companies only, as opposed to investments in listed shares and other investments such as bonds and money market instruments in the case of CISs.

Hedge Funds

A hedge fund (HF) is similar to a pooled fund such a unit trust and a retirement fund in that it takes in funds from investors and invests the funds on behalf of them in financial assets. However, it differs in that it has: less of the statutory limitations of the other collective investment schemes (i.e. pooled funds), a large relative proportion of funds taken in are forthcoming from the management company and the fund managers and, apart from being a "normal" investment vehicle (i.e. a "long only" investment vehicle), it is able to:

- Use leverage (i.e. borrow funds – apart from the funds of investors)
- Go “short” of securities
- Engage in derivative transactions

This institution is mentioned here for the sake of completeness. Individuals rarely invest in HFs

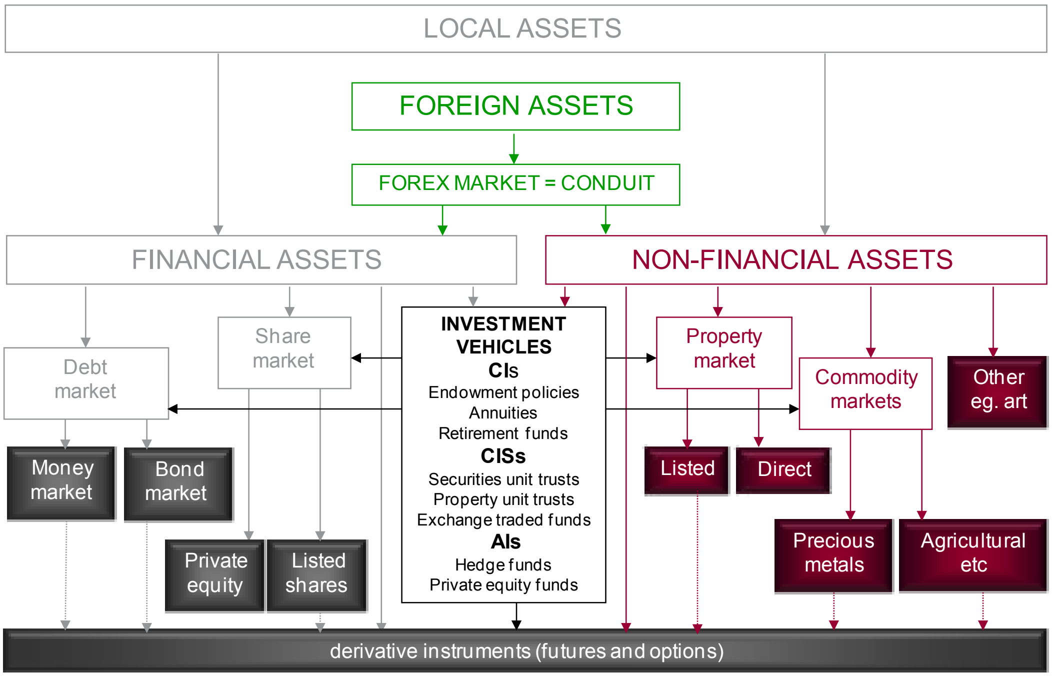
Foreign Investments

Foreign assets are comprised of the same asset classes as local assets. This is obvious because assets classes are the same worldwide. The difference between the asset classes in smaller countries and those in the larger economies is that the variety of assets in the larger economies is vast; in fact so vast that small country fund managers tend, in their foreign asset class selection, to rely on the expertise of foreign fund managers or invest in these markets via foreign investment vehicles.

There are many considerations in foreign investment selection, the most significant of which are currency risk and the diversification opportunities: individual investors would be wise to spread their foreign investments among a number of currencies in order to reduce risk

A summary of the investment markets is shown on the next page.

Figure: Summary of the investment markets



CHAPTER 5: FINANCIAL INSTRUMENTS: CALCULATION BASICS

Understanding the calculations for financial instruments is crucial when making investment decisions. The following section will cover an introduction to valuation focusing on:

- Money Market Instruments (Discount Instruments)
- Bonds (Coupon Paying Bonds)
- Foreign Exchange

5.1. Money Market Instruments

Money market instruments are of two forms, viz.:

- **Discount Instruments** are instruments which pay no explicit interest, are issued at a 'discount' to their face value. The interest earned is the difference between the purchase price and the face value at maturity. For e.g., Treasury Bill issued by the SA government.
- **Yield Instruments** are instruments which pay interest at a stated rate.

The calculation will focus only on **discount instruments**.

5.1.1. Definition of Terms

- N = Nominal Amount / Face Value = Amount of money that the investor will receive at maturity.
- D = Discount to Face Value
- d = Number of days in the discounting period / investment period
- P = Price paid for the instrument or Proceeds from the sale of the instrument
- i_d = Interest rate at which the instrument is valued
- B = Year basis = (actual number of days/365) for South Africa

5.1.2. Discount Instruments – Calculation

- Quotation basis = 'discount rate' – usually a percentage
- Calculation of the 'Rand' discount price given a 'discount rate:

$$P = N - D = N\text{x}\left[1 - \left(\frac{i_d}{100}\text{x}\frac{d}{B}\right)\right]$$

<u>Example</u>

Assume that you decide to tender for Treasury Bills at the weekly Friday SARB tender. It is a 91-day Treasury Bill with a 'discount rate' of 10.75% and a nominal amount/face value of R100. Calculate the tender price for this instrument.

<u>Calculation</u>

- N = Nominal Amount/Face Value = R100
- Discount rate = 10.75%
- d =Number of days in the discounting period = 91
- B = Year basis = 365

Using the above formula, we get:

$$P = N - D = N\text{x}\left[1 - \left(\frac{i_d}{100}\text{x}\frac{d}{B}\right)\right]$$

$$= 100\text{x}\left[1 - \left(\frac{10.75}{100}\text{x}\frac{91}{B365}\right)\right]$$

$$= \text{R}97.32$$

5.2. Bond Markets

Like money market instruments, bonds are of two forms, viz.:

- **Zero Coupon Bonds (Discount Instruments)** are instruments which pay no explicit interest, are issued at a 'discount' to their face value and have a fixed maturity. The interest earned is the difference between the purchase price and the face value at maturity.
- **Coupon Paying Bonds (Yield Instruments)** are instruments which pay interest at a stated rate.

The calculation will focus only on **zero coupon bonds**.

5.2.1. Definition of Terms

- FV = Face Value = Amount of money that the investor will receive at maturity.
- n = Term of the bond = usually stated in years
- P = Price paid for the instrument or Proceeds from the sale of the instrument
- *i* = Interest rate or bond yield
- C = Coupon/Interest payment

5.1.2. Zero Coupon Bonds – Calculation

Formula for the calculation of the price/value of a zero coupon bond:

$$P = \frac{FV}{(1+i)^n}$$

<u>Example</u>

Assume that you decide to buy a 5 year zero coupon bond with a face value of R100 and a yield of 6%p.a. what price would you be willing to pay for this bond?

<u>Calculation</u>

- FV = Face Value = R100
- *i* = Yield = 6% = 0.06 (in decimals)
- n = Term of the bond = 5

Using the above formula, we get:

$$P = \frac{FV}{(1+i)^n}$$
$$= \frac{100}{(1+0.06)^5}$$
$$= \text{R}74.73$$

5.3. Foreign Exchange Market

An **exchange rate** is how much it costs to exchange one currency for another. Exchange rates fluctuate constantly as currencies are actively traded. This pushes the price up and down, similar to other assets such as gold or stocks. The market price of a currency--how many Rands it takes to buy a US dollar for example--is different than the rate you will receive from your bank when you exchange currency.

The order in which the pair of exchange rates are listed (R/$ versus $/R) matters. Remember the first currency is always equal to one unit and the second currency is how much of that second currency it takes to buy one unit of the first currency.

5.3.1. Calculating Exchange Rates

Currently the R/$ exchange rate is R15.66, that means that it costs R15.66 to buy 1 US dollar.
If you wanted to know how many US dollars it costs to buy R1, use the following formula:

$$Cost\ of\ R1\ in\ US\ dollars = \frac{1}{\$/RExchange\ Rate}$$

In our example above, it means that R1 costs 0.06385 US cents, i.e., (1/15.66).

5.3.2. Calculating Conversion Spreads

When you go to the bank to covert currencies, you most likely won't get the market price that traders get. The bank or currency exchange house will mark-up the price so they make a profit.

If the $/R price is R15.66, the market is saying it cost R15.66 for $1. At the bank though, it may cost R15.75. The difference in the market exchange rate and the exchange rate they charge is their profit. To calculate the percentage discrepancy, take the difference between the two exchange rates, and divide it by the market exchange rate and multiply the result by 100 to get the percentage mark-up:

(15.75 – 15.66)/15.66 = 0.005747
0.005747 x 100 = 0.5747%

5.3.3. Calculation of Cross Exchange Rates

The idea of cross rates implies two exchange rates with a common currency, which enables you to calculate the exchange rate between the remaining two currencies. For example, you can calculate the £/$ exchange rate using the $/R and £/R exchange rates.

Currently the $/R exchange rate is R15.66/$ and the £/R exchange rate is R22.39/£. Therefore, the $/£ exchange rate is calculated as follows:

£/$ = 22.39/15.66 = 1.42

This means that it costs $1.42 to buy £1.

CHAPTER 6: DEFINITION AND OBJECTIVES OF INVESTMENT

The term *investments* refers to a portfolio of assets purchased with available funds that provides a return in the form of periodic cash flows and/or a gain (or loss) in the amount of the original amount invested (the capital). This tells us that there are two parts (either or both) to a return on an investment:

- a periodic cash flow
- a change in the value of the original investment (capital value), which may be positive or negative

Flowing from this, the *objective* of an investment is to *increase* the amount of the original investment by:

- earning a periodic cash flow and/or
- earning a gain in the value of assets (making a capital gain)

Assets need to be managed. Fund / portfolio management is the practice of asset allocation, i.e. the ongoing decision-making in respect of the allocation of funds between risky and non-risky assets, as well as choosing specific assets within asset classes. It is a balance between risk and return. The asset allocation function is based on in-depth asset market research.

Investment is not gambling. Gambling is a game of chance in which the probability of loss (= risk) is high. With investments, the probability of loss can be small because there are methods of investment management to reduce risk and enhance returns.

Investment is also not speculation. Speculation is investing own and/or borrowed funds for short-term periods (often intra-day), and the probability of profit is substantially higher than with a gamble. This is so because it is founded on research (technical and/or fundamental). However, the risk is lower than in gambling and higher than in long-term investing.

6.1 Risk-free rate

The risk-free rate (RFR) is a concept that occupies centre-stage in investments / finance. It is a concept that some scholars have difficulty in defining (some have even said that it does not exist). In our view there is not one RFR, but a series stretching from the one-day treasury bill (TB) rate to the 30-year rate on government bonds; "it" is simply the rates on government securities (treasury bills and government bonds), which are available daily (in efficient money and bond markets) and you can choose whichever rate you require as a benchmark for an investment.

What does this mean? It means that the RFR is the lowest rate that can be earned with certainty, and that you (when considering an investment for 5 years, for example) should regard the current 5-year bond rate as the minimum return you are willing to accept. It follows that every non-government, i.e. *risky*, investment should deliver a return [call it your *required rate of return* (RRR)] equal to the RFR plus a risk premium (RP):

$$RRR = RFR + RP$$

This simple formula should be the starting point when consideration is given to any investment.

What does risk-free mean? It means that if you purchase a government security, the rate at which it is bought is *certain to be earned*, and this is because governments don't default, however defaults do occur, but they are rare (since they have the authority to borrow and tax in order to repay and service their debt).

Thus, there are two broad investment categories: risk-free and risky assets / investments. *Risk-free* assets are government securities which deliver certain but lower returns. *Risky assets* are non-government securities (shares, corporate bonds, property, etc.) which deliver uncertain but higher returns (depending on the holding period). As we will show later, there is a positive relationship between return and risk.

However, it is important to mention that risk-free assets are only *credit-risk-free* – as said, because a government has the power to tax and borrow funds. They are not *market-risk-free* if they are sold before maturity. What does this mean? It means that the return is only certain if the asset is not traded in the secondary market. Market prices are opposite to market rates, and if the market rate rises to a higher level than the purchase rate, the price will be lower, and a capital loss will be made. However, this is irrelevant in the sense that the RFR just acts as a benchmark return.

6.2 Investment environment

What is the investment environment? The investment environment is the international economy and the domestic economy, developments in which have an effect on the values (prices) of the assets of the asset classes. It is well known that the prices of financial assets, particularly shares, can be extremely volatile, and this introduces the element of risk in financial markets. Investment risk is broadly defined as *volatility* in asset prices and it is measured in these terms (see later). Market prices / rates are volatile and this is the chief risk faced in financial / real asset markets and this takes place in the investment environment.

Ultimately, gross domestic product (GDP) growth is the major driver of asset prices, and asset price changes (positive and negative) are often worsened by the irrational behaviour of participants in the investment arena (known as the "herd instinct"). GDP is driven by gross domestic expenditure (GDE) and the trade account balance (TAB). GDE is driven by the consumption expenditure (C) and investment expenditure (I) of the private and government sectors, such that C + I = GDE. This is domestic demand. Foreign demand for local products is reflected in exports (X) while imports (M) reflect domestic demand for foreign goods. So, X – M = TAB = *net foreign demand*. The "big picture" (the entire economy) is complete:

$$C + I = GDE$$
$$GDE + TAB = GDP \text{ (= the total of expenditure on GDP)}$$

GDP is the total of net domestic production in a year, also called aggregate demand.

Interest rates are a significant factor in the economy and, therefore, the financial markets: they are the counterpart of certain asset prices (debt assets) and a significant input into the pricing of dividend-yielding shares and rent-yielding property. Short term rates (the lower end of the yield curve), as we have seen, are under the "control" of the central bank, the guardian of financial stability. They are the main instrument of central bank monetary policy, and exert a powerful influence on the bank lending rates, and therefore on the borrowing behaviour of the private sector, which drives money creation and GDP.

Money (M) = bank deposits (in the main; a small part is notes and coins), and changes (which largely are increases) in M (ΔM) are overwhelmingly caused by increases in domestic bank credit (DBC) extended (= the purchase of local financial assets) and the purchase of foreign financial assets (= foreign bank credit extended = FBC). Thus, ΔM is caused by the balance sheet causes of changes (BSCoC) as follows:

$$\Delta M = \Delta DBC + \Delta FBC^{2}$$

Underlying the BSCoC is a multitude of factors, including the actual demand for credit (DfC), interest rates which affect the DfC, the state of the economy and expectations regarding it, etc.

Money creation ΔM+ is a critical factor in GDP growth (ΔGDP) and according to the adjusted Fisher quantity theory of money (QTM) [V = velocity of circulation of money (generally a stable number); R = real = adjusted for inflation (P)]:

$$\Delta M + \Delta V = \Delta P + \Delta RGDP$$

This significance embodied in the QTM is that money growth is an essential ingredient in GDP growth, and that it is maximised if P is kept low (= a low and stable = predictable inflation environment) and this can only be achieved if the change in the demand for goods and services (= the demand side of GDP) (which underlies ΔM) is managed (inter alia by interest rates) at a level that can be satisfied by supply (= the production side of GDP).

So far, we have touched upon almost all of the essential elements of the investment environment. To them must be added the financial activities of government (= essentially the budget deficit), which results in borrowing in the financial sector. There are two main sources of funds: the holders of investment money

[2] Actually on a "net" basis, but we are keeping it simple here. ΔM = ΔDBC + ΔFBC, should be: ΔM = ΔnetDBC + ΔnetFBC, i.e. after the deduction of government deposits in the case of DBC and foreign deposits/loans in the case of FBC.

[mainly the "institutions" (= retirement funds, insurers, SUTs and ETFs) and money creation by the banks]. To the extent that the institutions' funds are accessed, the government "crowds out" the private sector, and to the extent that the banks buy government securities, money is created (ΔM+).
In summary, the essential elements of the investment environment = the macro economy, are the following:

- Δ(C + I).
- ΔTAB [expanded into the current account of the balance of payments (CaBoP) which includes other flows such as services payments/receipts, and its counterpart, the financial account of the BoP, the FaBoP].
- ΔM.
- Budget deficit.
- Interest rates (dominated by the central bank in the money market).

Why are shares the most volatile of all asset classes? It is because companies take on more risk (versus money market and bonds) in doing business (new projects, they are subject to the business cycle, etc.). Higher risk (measured as higher volatility) equates with higher return in the long term. Therefore history has generated data that demonstrates that shares have outperformed the other asset classes and that the asset classes have delivered returns in the following (descending) order:

- Shares (including hedge & private equity funds)
- Property
- Bonds
- Money market

For this reason, shares are the most sought-after financial asset, making this asset class subject to intense scrutiny (in the form of industry and company analysis), and susceptible to the herd instinct, captured in the new discipline “behavioural finance”. These influences make shares highly volatile in terms of price changes.

Given asset price volatility, fund managers or “investment houses” and broker-dealers, who service the fund managers; employ the services of investment analysts and specialist economists to anticipate future asset price developments.

6.3 Risk and return

6.3.1 What are risk and return?

We like return and dislike risk, but risk is ever-present in all financial markets, and there is a positive relationship between risk and return. In other words, risk and return are opposite sides of the same coin.

We know what return is: *capital gains / losses + income* (dividends or interest), and it is usually measured as holding period return[3] (HPR):

$$HPR = [(P_1 - P_0) + I] / P_0$$

where: P_0 = buying price; P_1 = selling price; I = income (in the form of dividends or interest).

If annualised (AHPR):

$$AHPR = (1 + HPR)^{1/n} - 1$$

where: n = years or fractions of a year (e.g. half a year will be 0.5).

But what is risk? It is the risk of the investment losing value (capital loss) or it is not yielding an income or both. This possibility is encapsulated in a measurable concept:

> *The probability of the actual return (HPR) on an investment being different from the expected return (E(R))*

[3] There are also other measures: arithmetic mean return, geometric mean return, internal rate of return.

There are two broad sources of risk (that contribute to the probability of HPR being different from E(R)):

- Security-specific risk (i.e. unsystematic risk)
- Market risk (i.e. systematic risk)

Security-specific risk arises from the activities of the specific companies, and the industry of which they are a part, and may be seen as the *major factors* that affect the *income flows* of companies. Analysts generally categorise this risk-type into *business risk* (examples: prolonged labour strike, arrival of serious competition from offshore, harmful management decisions, changes in product / service quality); *financial risk* (when debt is utilised as a source of capital, and is used injudiciously by the company); and *liquidity risk* (the risk of the segment of the share market in which the relevant share is being illiquid so that fair market value cannot be obtained).

Market risk is made up of the risks that are inherent in the financial and/or economic *system*. This risk affects all markets and little can be done about it. Examples of this type of risk are: tax changes, upward changes in interest rates (interest rate risk), political instability (country risk), the declaration of a war (country risk), a major change in the exchange rate (exchange rate risk), and a change in inflation (inflation risk).

6.3.2 Measuring risk and return

Measuring historical risk and return is straightforward, and it is best elucidated with an example using annual figures. Return over a year is HPR, and risk is the *standard deviation* of returns. The standard deviation is a measure of the *dispersion around the average return* (= the arithmetic mean) in percentage terms. The formula is:

$$\sigma^2 = \sum_{i=1}^{n} \frac{(x - M)^2}{n - 1}$$

where

σ^2 = variance of a set of values
x = each value in the set
M = mean (i.e. arithmetic average) of the set (mean return)
n = number in the set
σ = $(\sigma^2)^{1/2}$ (i.e. square root of σ^2) = standard deviation.

Table 1 shows the hypothetical HPR returns on a share for the years 1 to 4, and the relevant calculations.

Table 1: Calculation of historical standard deviation

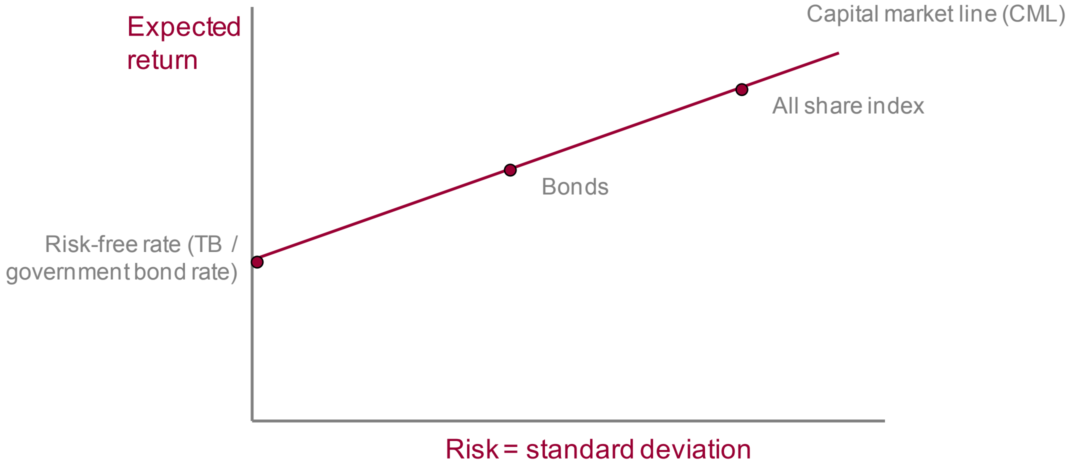
Year	HPR (%): X	X - M		$(X - M)^2$	
1	25	16.25		264.06	
2	15	6.25		39.06	
3	0	-8.75		76.56	
4	-5	-13.75		189.06	
	M = 8.75		$\Sigma\,(X - M)^2$ =	568.74	
			σ^2 =	568.74 / 3	= 189.58
			σ =	$(189.58)^{1/2}$	**= 13.77%**

This particular share has a *mean return* (M) of 8.75% and a *standard deviation* (σ) of 13.77%. It will be obvious that the higher the standard deviation, the higher the percentage dispersion around the mean, and, therefore, the higher the riskiness of this share.

6.3.3 Relationship between risk and return

The illustration below demonstrates the relationship between risk and return, and it is evident that the relationship is positive, i.e. the return required increases as risk increases. This is so because investors are *risk averse*. The relationship is represented by what is termed the *capital market line* (CML which is used extensively in portfolio literature). If investors were *risk seeking* (which would indicate a mental problem), the

CML would be negatively sloped. The slope of the CML depicts the extent of additional return expected / required for additional each unit of risk assumed.



Empirical evidence places assets classes at different positions in the above illustration of the risk and return relationship: money market at the bottom left, bonds in the middle and shares top right.

CHAPTER 7: BASICS OF PORTFOLIO MANAGEMENT

7.1. Investment Portfolio

An investment portfolio is a collection of assets owned by an individual or by an institution. Most investment portfolios, particularly portfolios that are assembled to pay for retirement, are made up mainly of securities, such as shares, bonds, mutual funds, money market funds and exchange traded funds. The best investment portfolios diversify the mix of investments -- which can range from the caution of SA Treasury bonds to the risky zip of small-company shares -- in an effort to dampen market losses and maximize potential gains.

7.2. Terminology

The following terms are used frequently in portfolio management and are very useful to know.

- **Portfolio** – a collection or grouping of investment securities or assets
- **Portfolio Return**
  … the expected return on a portfolio, k_p, is the weighted average of the expected returns on the individual stocks in the portfolio
  … the portfolio weights must sum to 1.0
  … the realized rate of return is the return that is actually earned on a stock or portfolio of stocks
- **Portfolio Risk**
  … the riskiness of a portfolio of securities, σ_p, in general is *not* a weighted average of the standard deviations of the individual securities in the portfolio
  … the correlation coefficient, r, is a measure of the degree of co-movement between two variables; in this case, the variable is the rate of return on two stocks over some past period
  … $-1.0 \leq r \leq +1.0$
  … the riskiness of a portfolio will be reduced as the number of stocks in the portfolio increases; the lower the correlation between stocks that are added to the portfolio the greater the benefits of continued diversification
- **Firm-Specific, Diversifiable, or Unsystematic Risk**
  … that part of a security's risk associated with random outcomes generated by events or behaviours specific to the firm; firm-specific risk can be eliminated by proper diversification
- **Market, Non-diversifiable, or Systematic Risk**
  … that part of a security's risk that cannot be eliminated by diversification because it is associated with economic or market factors that will affect most firms
  … the risk that remains after diversifying is market risk, or the risk that is inherent in the market, and it can be measured as the degree to which a given stock tends to "move" with the market
  … a stock's beta coefficient, β, is a measure of the extent to which the returns on a given stock move with the stock market as a whole (in most cases, a proxy for "the market" is used, such as the S & P 500 stock index)
  … by definition, the beta of the market, $\beta_M = +1.0$
- **Portfolio Beta**
  … the beta of a portfolio of securities, β_p, is a weighted average of the individual securities' betas

7.3. Portfolio Risk and Return Calculations

7.3.1. Calculating the Return on a Portfolio of Assets

Assume we formed a portfolio consisting of 2 shares. The return of the portfolio, (r_p) is the weighted average of the returns of the individual stocks, the weights being the proportions of their initially invested market values.

$$r_p = \sum_{i=1}^{2} w_i.(r_i)$$

where w_i is the market value weight of asset i and r_i is its return.

Example 1:

Calculate the return of the portfolio consisting of H and D stocks, given the market values of the individual stocks and at the time of investing and the returns of the individual stocks.

Shares	Investment (Rs)	Portfolio Weight	Share Return
H	1000	0.25	0.02
D	3000	0.75	0.03
Total	**4000**	**1.00**	

Return of the portfolio = .25(.02) + .75(.03) = 2.75%

7.3.2. Calculating the Risk of a Portfolio as Measured by Standard Deviation or Variance

The variance of a portfolio is a function of not only the variances of the individual assets within the portfolio but also of the covariances of returns among the assets.

We need to learn how to calculate the covariance between two assets first.

7.3.2.1. The covariance of returns between two assets

The covariance is the expected value of the product of the deviations of the returns of two assets from their respective mean values.

$$Cov(R_A, R_B) = \frac{\sum_{t=1}^{n}(R_{A,t} - \overline{R}_A)(R_{B,t} - \overline{R}_B)}{n}$$

where n is the number of periods in the sample, $R_{A,t}$ is the return of asset A in period t and $\overline{R}_A$ is the mean return for asset A.

The corresponding formula for the covariance between the returns of assets A and B is

$$COV(i,j) = \sum_{t=1}^{n}\left[r_{i,t} - E(r_i)\right]\left[r_{j,t} - E(r_j)\right]P_t$$

where i, j are two assets, t=1,.....,n are the range of possible states and P_t is the probability of state t occurring.

Example 2:

The conditional returns of stock I and J are forecast as follows. Calculate the covariance of their returns.

State of World	Probability of State	Return Share I	Return Share J
1	0.20	-0.18	-0.04
2	0.25	0.16	-0.02
3	0.30	0.12	0.21
4	0.25	0.40	0.20

$E(R_I) = 0.14$ $E(R_J) = .10$

Cov (I,J) = (-0.18-0.14)(-0.04-0.10)(0.2)+(0.16-0.14)(-0.02-0.10)(0.25)+ ....
= 0.0142

7.3.2.2. Calculating the variance of a portfolio

The variance of a portfolio is the sum of the variances of the individual assets and the sum of all the covariances between the assets, weighted by their market value weights.

$$VAR(p) = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j COV(i,j) \qquad i \neq j$$

Example 3: (using information from example 2)

If we assume our shares are in an equally weighted portfolio, i.e., each share makes up 50% of the portfolio, calculate the portfolio variance and standard deviation.

Variance of the portfolio = $(0.5)^2(0.193)^2 + (0.5)^2(0.116)^2 + 2(0.5)(0.5)(0.0142)$
= 0.01977

Standard Deviation of the portfolio = $\sqrt{Variance}$ = $\sqrt{0.01977}$ = 0.1406

7.4. Correlation and Its Impact on Diversification

7.4.1. Diversification of a Portfolio

Diversification is a risk-management technique that mixes a wide variety of investments within a portfolio in order to minimize the impact that any one security will have on the overall performance of the portfolio. Diversification lowers the risk of your portfolio without lowering the portfolio return.

Diversification of a portfolio can be achieved by combining different asset classes, for example, equities and bonds. Diversification can also be achieved by combining different investments within the same asset class, for example, within the asset class equities you could invest in Standard Bank and Murray & Roberts for the purposes of diversification.

The idea behind diversification is combining investments within a portfolio that respond differently to the same stimulus, for e.g., an increase in interest rates will be good for banks who lend money but bad for construction companies that borrow money.

Correlation is an important concept in the diversification of a portfolio.

7.4.2. Correlation

Correlation - a **correlation coefficient** tells you what percentage of two assets' price movements are driven by the same market forces. So, for example, if sunglasses and sunscreen are 100% correlated (having a coefficient of 1.00), their manufacturer's stock prices would move in the same direction 100% of the time. On the other hand, if sunscreen and umbrellas have a coefficient of -1.00 (negative 1.00), their prices always move in opposite directions, which means they move together 0% of the time. Correlation is also a standardized measure of covariance

$$\rho_{\mathrm{I,J}} = \frac{\mathrm{COV(I,J)}}{\sigma_{\mathrm{I.}}\sigma_{\mathrm{J}}}$$

Example 4:

If the covariance between assets I and J is 0.0142 and their standard deviations are 0.193 and 0.116 respectively, calculate the correlation coefficient.

$$\rho_{I,J} = \frac{COV(I,J)}{\sigma_{I.}\sigma_J} = \frac{.0142}{(.193)(.116)} = 0.63$$

The value of the correlation coefficient is within the bounds of +1 and -1, as indicated previously.

$1 > \rho > -1$

- If $\rho = 1$, the returns are perfectly positively correlated
- If $\rho = -1$, the returns are perfectly negatively correlated
- If $\rho = 0$, the returns are not correlated

For a portfolio to benefit from diversification, the correlation coefficient ρ must at least be *less than* +1. As ρ moves from +1 to -1 the benefits of diversification increase significantly. If you are able to find to assets where ρ *equals* -1, you are able to create a portfolio with a risk of 0.